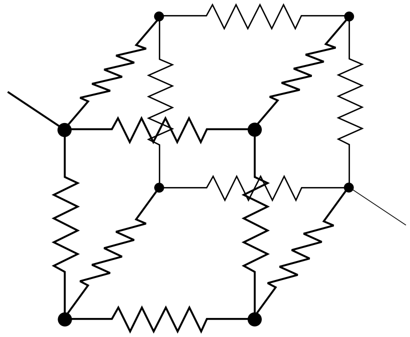
MISN-0-119

RESISTIVE D.C. CIRCUITS



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

RESISTIVE D.C. CIRCUITS

by

Peter Signell

Title: **Resistive D.C. Circuits**

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Input Skills:

1. Vocabulary: conductance, Ohm's law, resistance (MISN-0-118).
2. Calculate the power loss in a resistor, given its current and potential drop (MISN-0-118).
3. Use Ohm's law to calculate a resistor's current, resistance, or potential drop, given any two of those three quantities (MISN-0-118).

Output Skills (Knowledge):

K1. Vocabulary: circuit, electromotive force (emf), internal resistance, node, node-current rule, node-potential rule, parallel circuit, potential drop, resistor, series circuit.

K2. State the node-current and node-potential rules and explain the physical basis for each (i.e., explain how each results from a basic physical law).

K3. Derive the series and parallel resistance addition rules for resistors.

Output Skills (Rule Application):

R1. Apply the series and parallel resistance addition rules to calculate the effective resistance of a given resistor network.

Output Skills (Problem Solving):

S1. Given the resistances in a d.c. circuit, use simple rules to calculate the current at any node and the potential drop across, and power dissipation in, any circuit element.

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New authors, reviewers and field testers are welcome.

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1. Resistors and EMF'S

1a. Resistors. Resistors are pieces of conducting material used to establish desired values of current and potential difference in an electrical circuit. Figure 1 shows the symbol used to represent such a piece of conducting material.

1b. Direct Current Requires An Electromotive Force. For a direct current (that is, a steady-state current) to exist in an electrical circuit, the circuit must contain sources of energy, called sources of "electromotive force" (emf). Consider a circuit composed of just two resistors as in Figure 2. For the current i to exist as depicted, an electric field must do work against the "frictional" forces arising from collisions of the charges in the current with material in the resistors. We add the work done in the two branches of the circuit to get the total work done in moving the charges around the complete circuit. This is for a particular current over a particular period of time. Dividing the work by the time gives us the power that must be continually supplied to the circuit, the power that is continually being dissipated in the resistors in the form of heat. This power must be supplied by a power source, a "source of emf." There is no such power source shown in Figure 2 so a current cannot be sustained in that circuit. In Figure 3 we have added a power source and now the current can be sustained.

1c. Sources of Electromotive Force. Typical sources of emf include the electrochemical cell (for example, dry cell, lead-acid cell, nickel-cadmium cell), in which energy is supplied by chemical reactions, and generators, in which energy is supplied by mechanical work in moving a conductor through a magnetic field. Figure 3 shows a simple circuit with a power source, a "source of emf." The symbol for the source of emf is a short fat line parallel to a long thin line marked with a "+."

1d. Direction of the EMF. In a circuit diagram, the direction of an emf is, by convention, from a source of emf's short fat line to its long thin line. We indicate this by drawing an arrow in that direction, next to the source of emf, and we indicate the strength of the emf by the symbol $\mathcal{E}$. The arrow indicates the direction of flow of positive charges. Thus positive

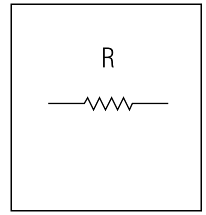


Figure 1. The symbol for a resistor.

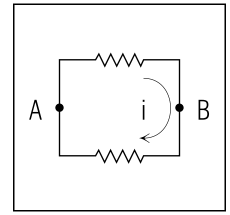


Figure 2. A closed electrical circuit.

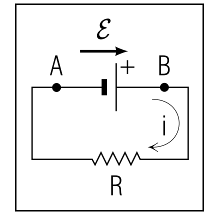


Figure 3. A circuit with a source of emf, $\mathcal{E}$.

charge will emerge from the long thin line, the "positive terminal" of the emf, and will flow into the short fat line, the "negative terminal." This is a battery, which contains a source of emf, usually has a "+" marked on one end.

1e. Units of EMF. The "strength" of a source of emf is specified as the amount of work per unit charge that the source will do in moving charge from the source's positive terminal to its negative one (around a circuit like the one in Figure 3). This "work per unit charge" done in moving a charge from one point to another is the definition of the potential difference of the two points. Thus the strength, $\mathcal{E}$, of a pure source of emf, is just the potential difference across its terminals. The common unit of emf is the volt, abbreviated V.

1f. Internal Resistance in Real Sources of EMF. A real source of emf, such as a flashlight battery, has a voltage drop across its terminals that decreases as the current it supplies increases. We usually simulate this effect by drawing a real source as a combination of an ideal source of emf plus an internal resistance as in Figure 4. By Ohm's Law, the voltage drop across the internal resistance is: $V_{\rm int} = iR_{\rm int}$. Thus the voltage measured at the terminals of a real source is:

$$V_{\rm terminals} = \mathcal{E} - iR_{\rm int}\,.$$

If $iR_{\rm int}$ is small compared to $\mathcal{E}$, the variation of voltage with "current drawn" will not be noticable and the source is called a "voltage source" (meaning a source of constant potential difference).

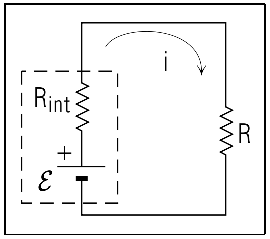


Figure 4. A Source of EMF with internal resistance.

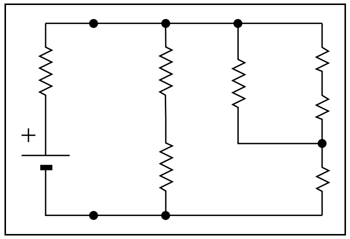


Figure 5. An example circuit.

2. The Circuit Rules

2a. Purpose and Definitions. We use Ohm's Law and two rules to determine the currents and voltages at various points in a simple circuit. By a simple circuit we mean one constructed of wires, resistors and a source of emf (Figure 5 shows an example). We also mean that two simple rules, to be developed in this section, can be used to find the voltage and current anywhere in the circuit. The wires are assumed to be perfect conductors and the resistors and the source of emf are connected to each other by such wires. The resistors and the source of emf are called "circuit elements." Points where wires from various elements meet are called "nodes." The two rules we use to analyze simple circuits concern nodes.

2b. The Node-Current Rule. The node-current rule is that the sum of all currents flowing into any node equals the sum of all currents flowing out of that node. This is true because we assume a steady-state exists so charge cannot accumulate anywhere. For example, if 5 coulombs of charge come into a node every second, then 5 coulombs of charge must leave that node every second.

2c. The Node-Potential Rule. The node-potential rule is that all directly-connected nodes are at the same potential. By directly-connected nodes, we mean those that can be reached from each other by traversing only wires. The rule holds even if there are other connecting paths containing circuit elements. The rule is true because we assume that the wires are perfect conductors, that it takes no work to transport charges through them. Since there is no work done, there is no potential drop and the nodes are at the same potential. Of course this is an approximation, but it is a good one if the resistances in the circuit elements are much

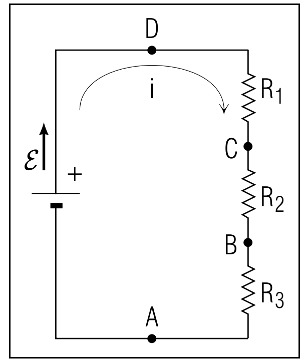


Figure 6. Resistors in a series circuit.

greater than any resistance offered by the wires.

3. Application to Simple Circuits

3a. Series Circuit. A "series circuit" consists of two or more resistors connected end to end, so that the current passes through each resistor in turn. The only nodes are those with one path in and one path out. Figure 6 depicts a circuit with three resistors in series. The node-current rule implies that the current must be the same in all parts of a series circuit.

3b. Series Addition Rule. The series addition rule determines the "effective resistance" of a set of series-connected resistors. The effective resistance is defined as that single resistance which can replace the set of resistors without changing the current. In a series circuit, the effective resistance is just the sum of the resistances in the set it is replacing. This seems reasonable because all charge must traverse all of them.

To demonstrate the series addition rule, look at Figure 6 and note that we will write $V_{DA} \equiv V_D - V_A$. By conservation of energy this may be expressed as:

$$V_{DA} = V_{DC} + V_{CB} + V_{BA}\,.$$

Using Ohm's law to express the potential drop across each resistor in terms of the current i, and the resistance of each resistor, we get:

$$V_{DA} = i\ R_1 + i\ R_2 + i\ R_3\,.$$

The effective resistance R produces the same current and obeys Ohm's Law, $i = V_{DA}/R$, so we combine this with the equation above to get:

$$R = \frac{V_{DA}}{i} = R_1 + R_2 + R_3.$$

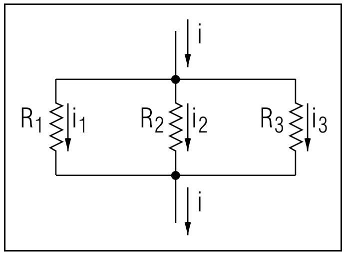


Figure 7. Resistors in parallel

In general, for N resistors in series,

$$R = R_1 + R_2 + \ldots + R_N = \sum_{n=1}^{N} R_n. \tag{1}$$

Equation (1) is the "series resistance addition rule."

3c. Parallel Circuit. A "parallel circuit" consists of two or more resistors that share the same points at which the current enters and leaves them, as illustrated in Figure 7. Applying the node-potential rule, the ends at which the current enters the elements are all at the same potential. Similarly, the ends at which the current leaves them are all at the same potential. Thus all of the resistors have the same potential drop.

3d. Parallel Addition Rule. The effective conductance for a set of parallel-connected resistors is just the sum of the conductances of the resistors in the set. To demonstrate this rule, look at Figure 7. By the node-potential rule, the various resistors have the same potential drop V. Let i_1 be the current flowing through R_1, i_2 the current flowing through R_2, etc. By the node-current rule, the total current i must equal the sum of the individual currents:

$$i = i_1 + i_2 + i_3.$$

Applying Ohm's law to each resistor separately, the above equation becomes:

$$i = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}.$$

Using $i = \dfrac{V}{R}$, we get for the effective resistance:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

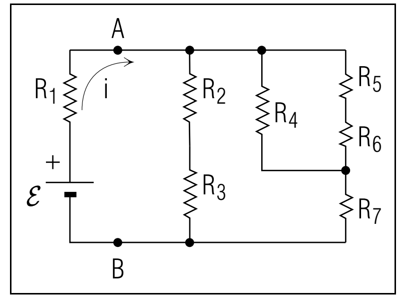


Figure 8. A series-and-parallel circuit.

For N resistors in parallel, the general result is:

$$\frac{1}{R} = \sum_{n=1}^{N} \frac{1}{R_n}. \tag{2}$$

Equation (2) is the "parallel resistance addition rule": the effective resistance of resistors in parallel is the reciprocal of the sum of the reciprocals of the individual resistances.

Since conductance G is the reciprocal of resistance, we can write Equation (2) as:

$$G = \sum_{n=1}^{N} G_n. \tag{3}$$

In summary, resistances add in series and conductances add in parallel.

Exercise: Show that, for the case of two resistors in parallel, the effective resistance is the product of the resistances divided by the sum of the resistances; that is,

$$R = \frac{R_1 R_2}{R_1 + R_2}. \tag{4}$$

3e. Series-and-Parallel Circuits. Complex resistive circuits can sometimes be treated as groups of series-connected and parallel-connected resistors. An example is shown in Figure 8. Our problem is to find the effective resistance of the circuit so we can determine the current it draws from any particular source of emf. We do this by alternately combining all series-connected and parallel-connected sub-circuits, replacing each (in turn) by effective resistors. This is illustrated in Figure 9. Follow it through and verify that the effective resistance of the circuit to the right of points A and B is:

$$R = \frac{(R_2+R_3)\left[\dfrac{R_4(R_5+R_6)}{R_4+R_5+R_6}+R_7\right]}{R_2+R_3+\dfrac{R_4(R_5+R_6)}{R_4+R_5+R_6}+R_7}.$$

If R_2 through R_7 are each 10.0 ohms, then $R \simeq 9.09$ ohms. If $\mathcal{E} = 1.50$ volts and $R_1 = 0.100$ ohm then:

$$i = \frac{\mathcal{E}}{R_1+R} = 0.163\,\mathrm{A}$$

and $V_{AB} = \mathcal{E} - i\,R_1 = 1.48$ volts.

4. The General Case

If a circuit does not consist of a hierarchy of purely series and parallel subcircuits, then the technique developed in this module will not be sufficient and one must use Kirkhhoff's two general rules. The technique for doing this is developed elsewhere.[1]

Acknowledgments

Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

Glossary

- **circuit**: a network of electrically active and passive devices connected by a conducting medium (usually metal wires).
- **direct current**: charge flow in a constant direction through a conductor, often abbreviated as d.c.
- **effective resistance**: for a specific network of resistors, that single resistance which can replace the network without changing the current.

[1]See "Kirkhhoff's Rules for General Resistive Circuits" (MISN-0-156).

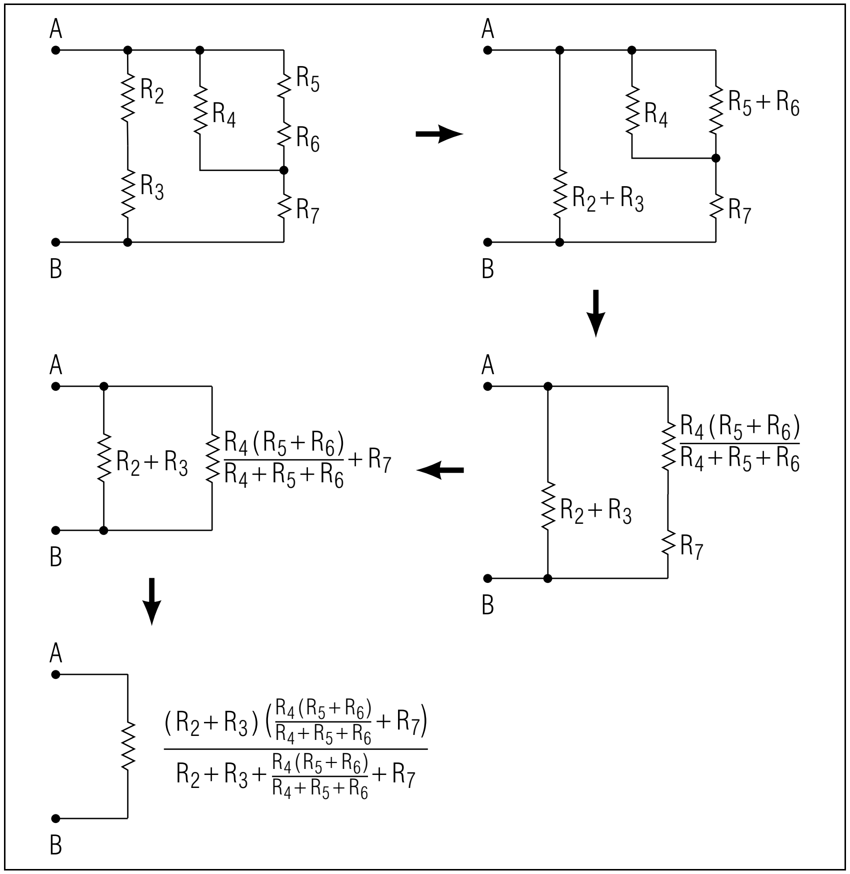


Figure 9. Successive replacement of sub-circuits in Fig. 8 by effective resistances.

- **emf (electromotive force)**: a source of energy that causes current to flow in an electrical circuit. Typical examples of emf's are dry cell batteries, car batteries, and mechanical generators.
- **internal resistance**: the resistance of a real source of emf to the flow of current through it (usually considered to be an unavoidable defect).
- **junction**: same as *node* (which see).
- **node**: a point in a circuit where current may take alternate paths in its flow.

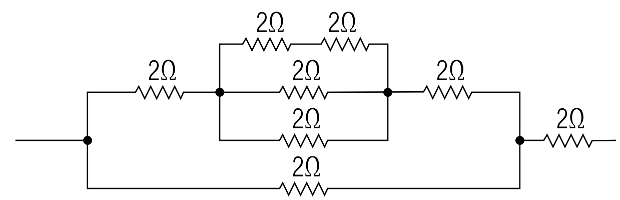
- **node-current rule**: a rule based on conservation of charge, stating that the current going into a node must equal the current going out of it.
- **node-potential rule**: a rule based on conservation of energy, stating that there is no potential drop between two nodes if no work need be done to transport charges between them (true for a perfect conductor, sufficiently true for the common connectors used in circuits).
- **parallel circuit**: a network whose circuit elements all share the same input and output points for current flow through them, so they all have the same potential drop.
- **resistor**: a piece of material used to provide a needed resistance in a circuit for the control of current and voltage in an electrical circuit.
- **series circuit**: a network of circuit elements arranged sequentially in a circuit so that charge must pass through each in turn.
- **potential drop**: the drop in electrical potential from one node to another in an electric circuit. The SI unit of potential drop is the volt.
- **voltage**: commonly used term for the electrical "potential drop" (which see) between two points when the *two* points in question are obvious.
- **voltage drop**: commonly used term for "potential drop" (which see).

PROBLEM SUPPLEMENT

Note: Problem 4 also occurs in this module's *Model Exam*.

Note: Work each problem *successfully* before going on to the next.

1. Determine the effective resistance of this network: *Help: [S-1]*

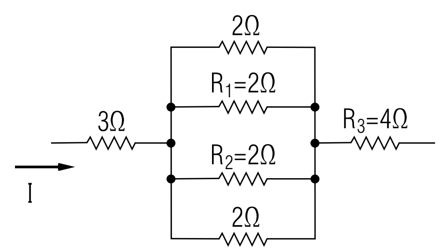


2. If resistor R_1 in the circuit shown below is dissipating heat at a rate of 18 watts find:

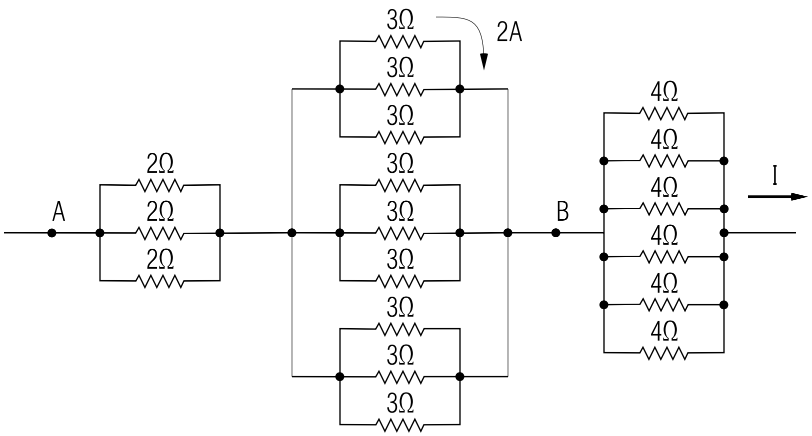
   a. the rate of heat dissipation by R_2; *Help: [S-9]*

   b. the rate of heat dissipation by R_3;

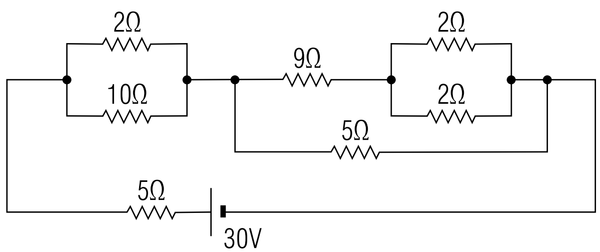
   c. the current I through the system.



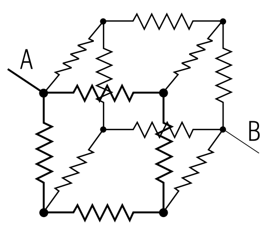
3. If a 2.0 A current flows in the branch of a circuit shown below what is the value of I? (Note: you should be able to write down the answer immediately without doing any work). What is the potential difference between points A and B? *Help: [S-2]*



4. For the circuit shown below calculate the current, power dissipation, and potential drop for the $2.0 \times 10^1\,\Omega$ resistor. *Help: [S-10]*



5. (only for those interested) Consider 12 resistors arranged along the edges of a cube as shown in the diagram below. If all the resistors are $1.00\,\Omega$, what is the effective resistance between A and B? *Help: [S-4]*



Brief Answers:

1. $(58/17)\,\Omega \simeq 3.4\,\Omega$
2. a. 18 watts; b. 576 watts; c. 12 amperes
3. 18 amperes, 18 volts
4. $(2/3)\,\mathrm{A} \simeq 0.67\,\mathrm{A}$; $(80/9)\,\mathrm{W} \simeq 8.9\,\mathrm{W}$; $(40/3)\,\mathrm{V} \simeq 13.3\,\mathrm{V}$
5. $(5/6)\,\Omega \simeq 0.83\,\Omega$

SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from PS, Problem 1)*

If you are having trouble with this problem, go back and reread the text, carefully working through the similar example given there. *Help: [S-3]*

S-2 *(from PS, Problem 3)*

If 2 A of current flows through the branch shown, and since all of the 3 Ω resistors have the same potential difference across them, what is the current in all of the other 3 Ω resistors? When you are able to answer this question correctly, use the node-current rule to get the current I.

S-3 *(from [S-1])*

Start by replacing the two series-connected resistors in the top-most branch by their single equivalent resistor. *Help: [S-5]*

S-4 *(from PS, Problem 6)*

If you utilize all the symmetries involved, the problem can be solved in a few lines. Assume a current i comes into node A and leaves node B. At node A the current has three alternative paths to take. Since each path is equivalent, the current splits into three equal parts. These three new currents each come to a new node with two equivalent alternate paths, so each of these currents splits in half. Now the six equal currents combine to give three equal currents which combine at node B to again yield a current i. Use these facts and Ohm's law to relate the *potential drops* along any simple path from A to B to the equivalent resistance.

We have found that in cases where students complained they still couldn't get the problem, they had not *really* tried doing *exactly* what the paragraph above described.

If you still can't get this problem, don't worry about it: it is intended only as an interesting exercise for those students who like the challenge.

S-5 *(from [S-3])*

Next replace the (now) three parallel-connected resistors down the upper center by their equivalent single resistor. *Help: [S-7]*

S-6 *(from [S-7])*

Next replace the (now) two parallel-connected resistors down the left side by their equivalent single resistor. *Help: [S-8]*

S-7 *(from [S-5])*

Next replace the (now) three series-connected resistors along the upper left branch by their equivalent single resistor. *Help: [S-6]*

S-8 *(from [S-6])*

Next replace the (now) two series-connected resistors by their equivalent single resistor and you are done.

S-9 *(from PS-problem 2a)*

"Heat dissipation," "power loss," and similar terms all mean the same thing. They are discussed elsewhere: see the *Input Skills* in this module's *ID Sheet*.

S-10 *(from PS-problem 4a)*

Work through and completely understand each paragraph and example in the module's *text*. Then you should be able to work this problem, and others, at will. If you still can't, at least you should be able to understand where the parts of this equation came from:

$$i_{20\,\Omega} = \frac{\left(\dfrac{30\,\mathrm{V}}{5\Omega + \dfrac{1}{\dfrac{1}{10\Omega}+\dfrac{1}{20\Omega}} + \dfrac{1}{\dfrac{1}{5\Omega}+\dfrac{1}{X}}}\right)\left(\dfrac{1}{\dfrac{1}{10\Omega}+\dfrac{1}{20\Omega}}\right)}{20\Omega} = \frac{2}{3}\,\mathrm{A}\,,$$

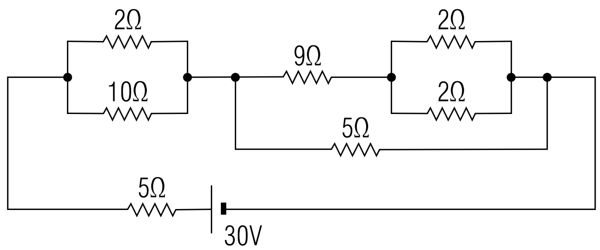
where:

$$X \equiv 9\Omega + \frac{1}{\dfrac{1}{2\Omega}+\dfrac{1}{2\Omega}}\,.$$

MODEL EXAM

1. See Output Skills K1-K3 in this module's *ID Sheet*.

2. For the circuit shown below determine the current, power dissipation, and potential drop for the $2.0 \times 10^1\,\Omega$ resistor. In solving, show each step in the reduction from the given resistor array to a single equivalent resistor.



Brief Answers:

Grader: To receive credit, the student must show, explicitly, each step in the reduction to a single equivalent resistor. Each step must have a correct circuit diagram with correct values for any remaining true resistors and for any "equivalent resistors."

1. See this module's *text*.

2. See this module's *Problem Supplement*, problem 4.