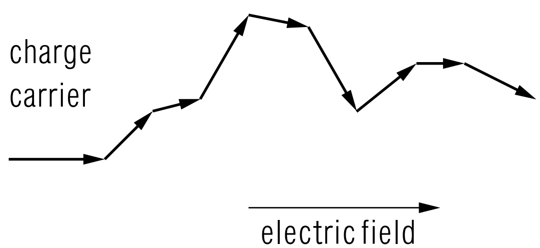
PHYSNET

MISN-0-118

CONDUCTIVITY AND RESISTANCE



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

CONDUCTIVITY AND RESISTANCE

by

Frank Zerilli

Title: **Conductivity and Resistance**

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Input Skills:

1. Vocabulary: ampere, current (MISN-0-117).
2. Explain how an electron can gain energy from an electric field (MISN-0-117).

Output Skills (Knowledge):

K1. Vocabulary: average thermal speed, conductance, conduction electrons, conductivity, conductor, current density, drift velocity, joule heating, lattice, mho, ohm, resistance, resistivity.

K2. Derive Ohm's law, starting from the motion of a conduction electron under the influence of both an electric field and a retarding force proportional to the electron's velocity.

K3. Give approximate values for the average thermal speed and drift velocity for a conduction electron.

K4. Explain how energy from an electric field is changed into heat in a resistor.

K5. Give two examples of non-ohmic systems.

Output Skills (Problem Solving):

S1. Given a wire or conducting cylinder of known length, diameter, and resistivity (or conductivity), calculate its resistance (or conductance).

S2. Given two of the four quantities associated with ohmic systems (current, voltage, resistance and power dissipation), calculate the other two.

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1. Ohm's Law

1a. General Statement of Ohm's Law. In most solid materials at constant temperatures, an applied electric field causes an electrical current to flow which is proportional to the magnitude of, and in the same direction as, the field. This relationship is known as Ohm's law.[1] Metals obey Ohm's law very accurately for electric fields which are much less than 10^8 volts/meter in magnitude.

1b. Current Density vs. Electric Field. Ohm's law may be written in terms of "current density" and electric field strength. Current flow in a conducting material may be described in terms of current density $\vec{j}$, which is the current flow per unit cross-sectional area perpendicular to the direction of charge flow (see Fig. 1). In terms of current density, the statement of Ohm's law is:

$$\vec{j} = \sigma \vec{E}\,, \tag{1}$$

where $\vec{E}$ is the applied electric field and σ is a constant, characteristic of the material, called the "conductivity." Eq. (1) relates the current density at a point in the material to the electric field at that point.

1c. Current vs. Potential Difference. Ohm's law may also be expressed in terms of current and potential difference. Consider a cylindrical conducting material of length ℓ and cross-sectional area A (a "wire") in which a steady current i is flowing due to a uniform electric field $\vec{E}$ along

[1] After George Simon Ohm (1787-1854), a German physicist.

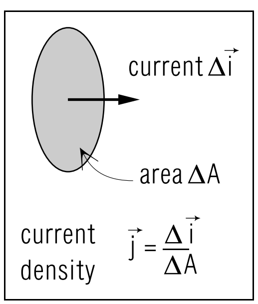


Figure 1. The definition of current density.

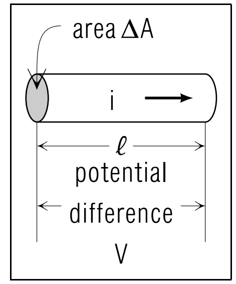


Figure 2. A cylindrical conductor under a potential difference.

the length of the conductor (see Fig. 2). The potential difference between the two ends of the conductor is $V = E\ell$ and the magnitude of the current density is $j = i/A$, so Ohm's law can be written

$$V = i\,R\,, \tag{2}$$

where $R = \ell/(\sigma A)$ is a constant called the "resistance." Whereas conductivity is a characteristic of a given material (like specific gravity is characteristic of a material), resistance is a property of a given object, determined not only by the material but also by the structure into which it is formed (analogous to the mass of an object).

1d. Resistivity, Resistance, Conductivity, Conductance. The reciprocal of conductivity, σ, is called "resistivity," ρ:

$$\rho = \frac{1}{\sigma}\,. \tag{3}$$

The reciprocal of resistance, R, is called "conductance," G:

$$G = \frac{1}{R}\,. \tag{4}$$

The MKS unit of resistance is the volt/ampere which is given the name "ohm," symbolized by Ω. The unit of conductance is the ohm^{-1} which is given the name "mho" ("ohm" spelled backward).

▷ Show that the units of resistivity are ohm-meters. *Help: [S-1]*

Table I shows some resistivities of typical metals and insulators.

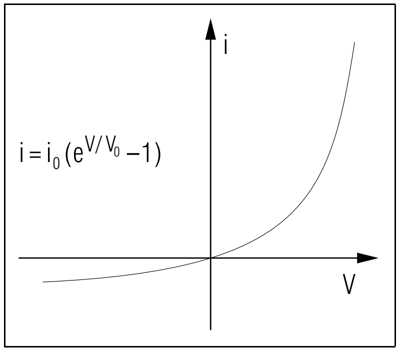


Figure 3. Current vs. voltage across a p-n junction.

Table I. Some Resistivities	
Material	Resistivity at 20 °C (ohm-cm)
Aluminum	2.8×10^{-6}
Copper	1.7×10^{-6}
Silver	1.6×10^{-6}
Iron	10×10^{-6}
Bismuth	120×10^{-6}
Sulfur	1×10^{17}
Maple	3×10^{10}
Slate	1×10^{8}
Plate Glass	2×10^{13}

1e. Non-Ohmic Systems. Although many solid materials obey Ohm's law very accurately over a wide range of conditions, there are many examples of systems in which Ohm's Law is not valid. For example, when a current flows through an ionized gas the relation between voltage and current is highly non-linear, that is, the ratio V/i is not a constant but varies with the value of i. The same is true for current flowing through the junction of a p-doped semiconductor and an n-doped semiconductor.[2] Figure 3 shows the non-linear relation of current to voltage in a p-n junction. Another example is the non-linear relation of current to voltage in a radio vacuum tube. Still another example is the tungsten filament incandescent lamp. The current which flows through the tungsten filament of the lamp causes significant temperature changes in the material. Since the resistivity of the tungsten increases with temperature, an increase in

[2]Transistors and diode rectifiers utilize p-n junctions.

current leads to an increase in the resistance of the filament.

2. Derivation of Ohm's Law

2a. Definition of Drift Velocity. The charge carrier drift velocity in a conducting material is proportional to the applied electric field strength. We can picture a conducting material as a "lattice" of atoms within which charge carrying particles (conduction electrons) move with a thermal distribution of velocities, every so often colliding with defects in the lattice. In the absence of an external electric field, the average thermal velocity of the charge carriers is zero since every direction of motion is equally likely. The average thermal speed (magnitude of the average thermal velocity) is not zero. For the conduction electrons in copper, the average thermal speed is about 1.6×10^6 m/s. When an external electric field is applied, the electrons are accelerated by the field producing an average "drift velocity" in the direction opposite to the field.[3] The effect of random collisions between the charge carriers and defects in the lattice produces an effective "frictional drag" force which is proportional to the drift velocity. Thus the drift velocity approaches a limiting velocity at which the frictional drag force balances the force due to the electric field. An analogy useful in understanding why the collisions lead to a limiting drift velocity is the case of a marble rolling down a staircase.[4] The gravitational field accelerates the marble, but the kinetic energy gained by the marble is given up in collisions with the steps (imagine that there is zero rolling friction, but that the collisions are inelastic), so that the marble rolls down the stairs at approximately constant average speed. The time it takes to approach the limiting drift velocity is of the order of the mean time between collisions (about 10^{-14} s in copper). The expression for the frictional drag force is

$$\vec{F}_{\rm drag} = k\vec{v}_D\,, \qquad (5)$$

where k is a constant, and the force due to the electric field is

$$\vec{F}_{\rm field} = e\vec{E}\,, \qquad (6)$$

[3]Since electrons have a negative charge, they are accelerated in the direction opposite to the field. However, the transport of negative charges in one direction is equivalent to the transport of positive charges in the opposite direction and, by convention, the direction of current flow is the direction in which the equivalent positive charge is transported, so that the direction of current flow is the same as the field direction.

[4]See D. Halliday and R. Resnick, *Physics* (Wiley and Sons, 1966) chapter 31.

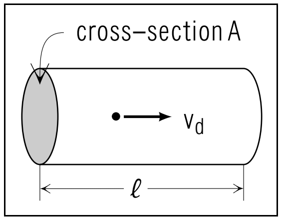


Figure 4. A cylindrical element of conducting material.

where e is the charge of the carrier, so by equating the two we find that the limiting drift velocity is proportional to the electric field:

$$\vec{v}_D = \frac{e}{k}\vec{E}\,. \tag{7}$$

2b. Relating Drift Velocity to Current Density. Current density $\vec{j}$ is proportional to the charge carrier drift velocity $\vec{v}_D$ and to the number of charge carriers per unit volume n:

$$\vec{j} = n\,e\,\vec{v}_D\,, \tag{8}$$

where e is the charge of the carrier. To derive this relation, consider a cylindrical element of conducting material of length ℓ and cross-sectional area A (see Fig. 4) containing n charge carriers per unit volume, moving with drift velocity $\vec{v}_D$ to the right. The time it takes one charge to move a distance ℓ is:

$$t = \frac{\ell}{v_D}\,. \tag{9}$$

Thus, in a time t, all the charge Q initially contained in the cylindrical element will have passed out the right end. The charge Q is the amount of charge per unit volume, ne, times the volume of the cylindrical element, $A\ell$. Therefore the current is

$$i = \frac{Q}{t} = \frac{neA\ell}{t} = nev_D A\,,$$

and the current density is:

$$j = \frac{i}{A} = nev_D\,.$$

▷ What is the drift velocity of the conduction electrons in a copper wire of 1 mm diameter carrying a current of 1 ampere? There are 8.5×10^{22} conduction electrons/cm^3 in copper. Answer: 9.3×10^{-5} m/s. *Help: [S-7]*

2c. Ohm's Law. Since current density j is proportional to the drift velocity v_D of the charge carriers [Eq. (8)] and the drift velocity is proportional to the applied electric field [Eq. (7)], we have derived one form of Ohm's law, $j = \sigma E$, where $\sigma = ne^2/k$.

2d. Validity of Ohm's Law. It is reasonable to assume that, when a charge carrier undergoes a collision with a lattice defect the velocity of the charge carrier is changed in a random manner. Thus any memory of the previous state of motion of the charge is lost. The average drift velocity of the carriers in the presence of the external electric field is then, simply, the velocity gained by the acceleration of the electric field in a time τ equal to the average time between collisions. If m is the mass of the charge carriers, then

$$\vec{v}_D = \vec{a}\tau = \frac{q\tau}{m}\vec{E}\,.$$

Thus Ohm's law is valid as long as the mean time between collisions does not depend on E.

3. Power Dissipation In Current Flow

3a. The General Expression. We wish to derive a general expression for the power dissipated when a current flows through a potential difference. Consider a "black box" (an arbitrary system) as shown in Fig. 5 with a current i flowing through it. Suppose the left end of the box is at a potential V and the right end is at zero potential. In a time Δt, an amount of charge $\Delta Q = i\Delta t$ passes through the box. It goes in with potential energy $V\Delta Q$ and comes out with potential energy zero. Since the kinetic energy has not changed,[5] the charge carriers have done work on something in the box. The power (rate at which work is done) is

$$P = \frac{V\Delta Q}{\Delta t} = V i\,. \tag{10}$$

[5] The charge carriers come out with the same distribution of velocities they had when they went in.

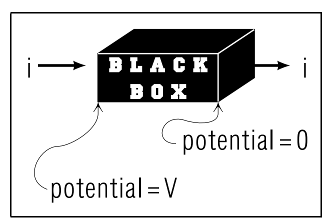


Figure 5. Power dissipation in an arbitrary system.

This is a general result and is merely a statement of conservation of energy.[6]

▷ Show that power dissipated per unit volume equals Ej where j is the magnitude of the current density and E is the magnitude of the electric field. *Help: [S-2]*

3b. Where the Dissipated Power Goes. What happens to the power dissipated depends on what is in the black box. If it contains a solid conducting material, the energy gets converted into heat energy. The work that the electric field does in accelerating the charge carriers is transferred to the atomic lattice when the charge carriers collide with the lattice defects. This increases the thermal energy of the lattice. This type of heating is called "joule heating." If the box contains a "storage battery" the energy gets converted to chemical energy via electrochemical reactions. On the other hand, the box might contain a motor which does mechanical work.

3c. If Ohm's Law Holds. If the box contains a system for which Ohm's law is valid, then Eq. (10) can be written in two alternate forms by using Ohm's law (Eq. (2) to eliminate either V or i from Eq. (10):

$$P = V\, i\,,$$

but

$$V = i\, R\,,$$

so

$$P = i^2\, R\,, \tag{11}$$

or

$$P = \frac{V^2}{R}\,. \tag{12}$$

Acknowledgments

Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

[6]See "Particle Energy in an Electrostatic Field" (MISN-0-117).

Glossary

- **average thermal speed**: the average speed of randomly moving charge carriers which move, with a thermal distribution of velocities, within a conductor.
- **conductance**: the property of a system that specifies the ease with which charge carriers pass through the system. Technically, conductance is the mathematical reciprocal of resistance. It is not a characteristic property of a given material but of a given object since it depends on the amount and shape of the conducting material in the object.
- **conduction electrons**: the electrons of a material that are responsible for current flow; these are the charge carriers. In a conductor these electrons are not associated with a single atom, but with the entire conducting material.
- **conductivity**: a characteristic property of a given material that specifies the current flow in the material for a given applied electric field. Mathematically, conductivity is the reciprocal of resistivity. It depends on the material composing the conducting object but is independent of the size or shape of the object.
- **conductor**: a material that allows charge carriers to pass through it. A material that is a poor conductor is usually called an "insulator."
- **current density**: a vector quantity whose magnitude at a point in space is the current per unit area normal to the direction of current flow, and whose direction is the direction of current flow.
- **drift velocity**: the average velocity, in the direction of the current, of the charge carriers in a conductor subjected to an electric field.
- **joule heating**: the process whereby heat is produced by the inelastic collision of conduction electrons with the lattice defects in a conductor. The energy transferred to the lattice causes the lattice atoms to vibrate more violently, raising the thermal energy of the lattice.
- **lattice**: a three-dimensional array of atoms or ions held together by chemical bonds (e.g. ionic, covalent, or metallic bonds).
- **mho**: a unit of conductance, equal to an ohm^{-1} (an inverse ohm). "mho" is "ohm" spelled backward.

- **ohm**: a unit of resistance, defined by Ohm's law as a resistance that causes a one-ampere current to drop in potential by one volt; denoted by the symbol Ω.

- **resistance**: the property of a system that specifies the difficulty with which the system passes charge carriers. The numerical value of the resistance R of an object is usually obtained by measuring the current I produced through the object by a known voltage V placed across the object: then $R = V/I$. Resistance is not a characteristic property of the object since it depends on the object's size and shape. Resistance is the reciprocal of conductance.

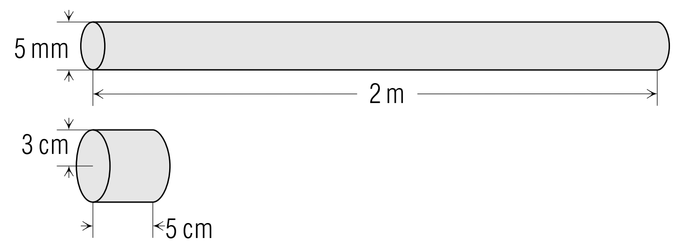
- **resistivity**: a characteristic property of a material that specifies the material's current-carrying capabilities, irrespective of the size and shape of the material. Resistivity is the reciprocal of conductivity.

PROBLEM SUPPLEMENT

Note: Problems 3 and 4 also occur in this module's *Model Exam*.

1. Assuming that there is one conduction electron per atom in copper, calculate the number of conduction electrons per unit volume for copper. The density of copper is 8.94 grams/cm^3 and its atomic mass is 63.54 amu. Note: 1 amu(atomic mass unit) = 1.657×10^{-27} kg.

2. A rod and a disc pictured below are composed of the same material and are at the same temperature. If the end-to-end resistance of the rod is 2.0×10^{-3} ohm, what is the resistance between opposing round faces of the disc? The rod is 2.0 m long and 5.0 mm in diameter. The disc is 5.0 cm long and 3.0 cm in radius.



3. A 10.0 ohm resistor produces heat at a rate of 2.5×10^2 watts. What current is flowing through it? What is the voltage across the resistor? What current would produce heat at half the present rate?

4. At (2.0×10^1) °C, copper has a resistivity of 1.7×10^{-6} ohm-cm. What is the resistance of a 1.0 meter length of copper wire that has a diameter of 1.0 mm?

5. If a potential difference of 1.2 volts exists between the ends of the wire in Problem 4, what current is flowing in the wire?

6. How much current flows through the filament of a 36 watt automobile headlamp that is connected to a 12 volt battery?

7. A current of 0.10 ampere flows through a conductor whose resistance is 47 ohms. What is the power dissipated in the conductor?

8. A potential difference of 15 volts exists across the ends of a conductor whose resistance is 22 ohms. What is the power dissipated in the conductor?

Brief Answers:

1. 8.53×10^{22} conduction electrons/cm^3 *Help: [S-3]*
2. 3.5×10^{-7} ohm *Help: [S-4]*
3. 5.0 amperes; 5.0×10^1 volts; for half power dissipation, 3.5 amperes
4. 0.022 ohm *Help: [S-5]*
5. 5.5×10^1 amperes *Help: [S-6]*
6. 3.0 amperes
7. 0.47 watt
8. 10.2 watts

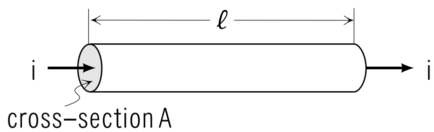
SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from TX-1d)*

resistance = length×resistivity/area [see Eq. (2) of module text] so MKS units of *resistivity* are:

$$\text{unit of resistivity} = \frac{\text{meters}^2 \times \text{ohms}}{\text{meters}} = \text{ohm-meters}$$

S-2 *(from TX-3a)*



If the potential difference across the ends of the conductor shown in the drawing is V, then $P = Vi$. But $V = E\ell$ and $i = jA$ so
$P = EjA\ell = Ej\times$ volume, so: power/volume $= Ej$

S-3 *(from PS, Problem 1)*

mass of one copper atom = $63.54(1.657 \times 10^{-24}\,\text{grams})$ and $1\,\text{cm}^3$ of copper = 8.94 grams so there are

$$\frac{8.94\,\text{grams}}{63.54 \times 1.65 \times 10^{-24}\,\text{grams}} = 8.53 \times 10^{22}\,\text{atoms}$$

in one cm^3 of copper; so if there is one conduction electron per atom, then there are 8.5×10^{22} conduction electrons per cm^3.

S-4 *(from PS, Problem 2)*

$$\frac{R_{\text{disc}}}{R_{\text{rod}}} = \frac{\ell_{\text{disc}}}{\ell_{\text{rod}}} \cdot \frac{A_{\text{rod}}}{A_{\text{disc}}} = \frac{\ell_{\text{disc}}}{\ell_{\text{rod}}} \cdot \left(\frac{r_{\text{rod}}}{r_{\text{disc}}}\right)^2$$

$$R_{\text{disc}} = \frac{5 \times 10^{-2}\,\text{m}}{2\,\text{m}} \cdot \left(\frac{\dfrac{5 \times 10^{-3}\,\text{m}}{2}}{3 \times 10^{-2}\,\text{m}}\right)^2 \cdot 2 \times 10^{-3}\,\text{ohms}$$

S-5 *(from PS, Problem 4)*

$$R = \frac{\rho\ell}{A} = \frac{(1.7 \times 10^{-6}\,\text{ohm} \times 10^{-2}\,\text{m}) \cdot (1\,\text{m})}{\pi \left[\dfrac{1 \times 10^{-3}\,\text{m}}{2}\right]^2}$$

S-6 *(from PS, Problem 5)*

$$I = \frac{V}{R} = \frac{(1.2\,V) \cdot \pi \left[\dfrac{1 \times 10^{-3}\,\text{m}}{2}\right]^2}{(1.7 \times 10^{-6}\,\text{ohm} \times 10^{-2}\,\text{m}) \cdot (1\,\text{m})}$$

S-7 *(from TX-2b)*

The answer is the quotient of a numerator and a denominator. The numerator is:

$$1\,\text{A} = 1\,\text{C/s}$$

and the denominator is the product of these three expressions (with the value of the electron charge taken from this volume's *Appendix*):

$$(\pi)(0.5\,\text{mm})^2(10^{-3}\,\text{m/mm})^2$$
$$(8.5 \times 10^{22}\,\text{electrons/cm}^3)(10^2\,\text{cm/m})^3$$
$$(1.602 \times 10^{-19}\,\text{C/electron})$$

MODEL EXAM

1. See Output Skills K1 - K5 in this module's *ID Sheet*. The exam may have one or more of these skills, or none.
2. At 20 °C, copper has a resistivity of 1.7×10^{-6} ohm cm. What is the resistance of a 1.0 meter length of copper wire of 1.0 mm diameter?
3. A 10.0 ohm resistor produces heat at a rate of 2.5×10^{2} watts. What current is flowing through it? What is the voltage across the resistor? What current would produce heat at half the present rate?

Brief Answers:

1. See this module's *text*.
2. See Problem 4, this modules's *Problem Supplement*.
3. See Problem 3, this modules's *Problem Supplement*.