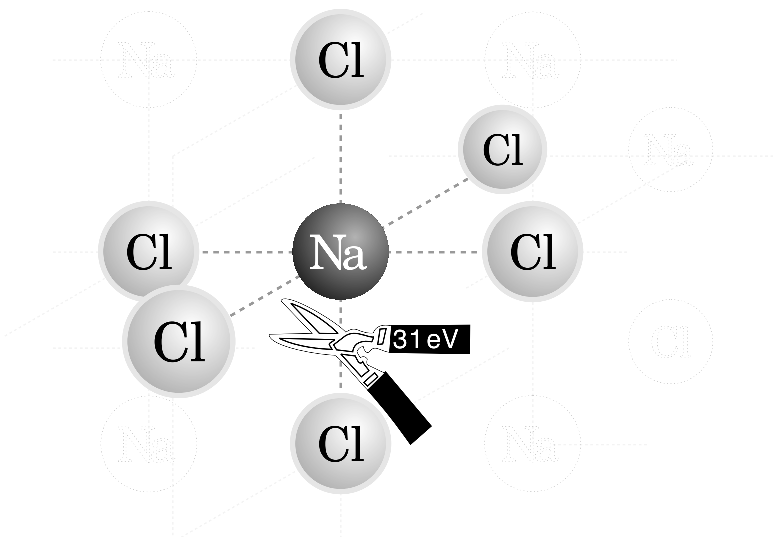
MISN-0-116

THE ELECTROSTATIC POTENTIAL



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

THE ELECTROSTATIC POTENTIAL

by

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Title: **The Electrostatic Potential**

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Input Skills:

1. Vocabulary: conservative force field (MISN-0-21).
2. State the expression for the electrostatic force between two charged particles (MISN-0-114).
3. Write down the expression for the electric field of a charged point particle (MISN-0-115).
4. Calculate the work done by a force along a given path (MISN-0-21).
5. Use the definition of potential energy to determine the potential energy function associated with the conservative Coulomb force field (MISN-0-21).

Output Skills (Knowledge):

K1. Vocabulary: electrostatic (electric) potential, volt.

Output Skills (Problem Solving):

S1. Calculate the electric potential at a given point in space due to a given distribution of discrete point charges (one or more).

S2. Calculate the electric potential energy of a charged particle placed at a point in space where the potential is known.

S3. Calculate the total potential energy of a given collection of point charges. Using the work-energy principle, determine where this energy goes when the charged particles rearrange.

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1. Introduction

1a. Overview. In this module we build on the idea of the electrostatic field to develop the idea of *electrostatic potential*, which is the capacity of an electrostatic field to do mechanical work. This is an important concept for designing or repairing almost any electric or electronic device. The strength of electrostatic potential is measured in volts, usually using a *voltmeter*, and is often referred to as "the voltage." For example, "The voltage on this Walkman battery is down to 8.2, where it should be at least 8.8 volts." The electrostatic field is a vector quantity but the electrostatic potential is a scalar quantity. We will find that for any electrostatic field which is specified throughout a region, we can perform a scalar (one-dimensional) integration on it to get the electrostatic potential throughout the region. Similarly, given the electrostatic potential throughout a region, we can take a vector (three-dimensional) derivative of it to obtain the electrostatic field throughout the region. As an intermediate concept between the electrostatic field and the electrostatic potential, we will deal with the *electrostatic potential energy* of specific charged particles.

1b. The Gravitational Analogue. Recall that gravitational potential energy is a very useful concept. For example: If an object is dropped, then its gravitational potential energy is converted to kinetic energy and its speed can be easily calculated. One obtains an object's gravitational potential energy by calculating the work that must be done against the force of gravity while one mechanically moves the object from some reference point to the object's present position. For example, if you take an object with a mass of 5 kg from ground level to the top of a 30 m tall building, then the potential energy of the object increases by 150 J. If the object is then dropped from the top of the building down to the ground level, the 150 J of gravitational potential energy at the top is converted to kinetic energy at the bottom plus some energy that went into frictional heating of the the air through which the object passed as it descended. Thus the gravitational potential energy at the top equals the work done, against gravity, in taking the object from ground to top, and it also equals the work done by gravity in taking the object from top to ground. One

word of caution: When doing electrostatic calculations and making analogies to the gravitational case, note that masses always experience a force in the direction of the gravitational field. Thus for the electrostatic case the equivalent of a mass is a positive charge since it always experiences a force in the direction of the electrostatic field.

2. Electrostatic Potential Energy and Potential

2a. Electrostatic Potential Energy. If a particle having charge q is placed at a space-point $\vec{r}$ in a region where an electrostatic field $\vec{E}$ exists, then the electrostatic field exerts an electrostatic force $\vec{F}$ on the charged particle:

$$\vec{F} = q\vec{E}\,. \tag{1}$$

If the charged particle is mechanically moved along some path from one space-point to another space-point, the total work done *against* the electrostatic force is:[1]

$$W(q, \vec{r}_1 \to \vec{r}_2,\ \text{against } \vec{E}) = \int_{\vec{r}_1}^{\vec{r}_2} (-\vec{F}) \cdot d\vec{\ell} = -q \int_{\vec{r}_1}^{\vec{r}_2} \vec{E} \cdot d\vec{\ell}, \tag{2}$$

where $d\vec{\ell}$ is an element of length along the path. The electrostatic force is known to be a conservative force, so the work in Eq. (5) is the same for all paths connecting the same two end-points and that means we can define a potential energy $E_p(q, \vec{r})$ for a particle with charge q at space-point $\vec{r}$ without saying what path was used to get there:[2]

$$E_p(q, \vec{r}_2) - E_p(q, \vec{r}_1) = W(q, \vec{r}_1 \to \vec{r}_2,\ \text{against } \vec{E})\,. \tag{3}$$

Note that only *changes* in potential energy have been defined. However, we can stop talking about changes between two points in space if we specify some point $\vec{r}_0$ as the electrostatic potential energy *reference point*,

[1] The definition assumes that a bare minimum of work is done, just enough to barely overcome the electrostatic force and not, for example, enough to change the particle's kinetic energy. The force necessary to barely overcome the electrostatic force is just the negative of the electrostatic force.

[2] (only for those interested) We know both experimentally and theoretically that the electrostatic force is a *conservative* force. The theoretical proof uses a remarkable mathematical theorem which says that since $\vec{\nabla} \times \vec{F} = 0$ for all electrostatic forces, a line integral of $\vec{F}$ between any two points is independent of the particular path used for the line integral. The line integral of the force is just the work done, so the work done is path-independent and that means the force is conservative.

a point where we specify that the electrostatic potential energy is zero:[3]

$$E_p(q, \vec{r}_0) = 0\,.$$

Then we can talk about the particle's actual potential energy at any point, although it is to be understood that we are referring to the change in potential energy when the particle is moved from the reference point to the point in question:

$$E_p(q, \vec{r}) \equiv W(q, \vec{r}_0 \to \vec{r},\ \text{against } \vec{E})\,.$$

2b. Electrostatic Potential. In dealing with electrostatic devices, it is often useful to deal with a given electrostatic field's ability to transfer stored energy to a wide variety of amounts of charge. Thus at any space-point $\vec{r}$ we define the *electrostatic potential*, $V(\vec{r})$, as the work *per unit charge* that must be done against the electrostatic field to bring any charge from the reference point to the space-point in question. Equating the work-per-unit-charge to potential energy-per-unit-charge, we get the electrostatic potential, $V(\vec{r})$, as:

$$V(\vec{r}) \equiv \frac{E_p(q, \vec{r})}{q} = -\int_{\vec{r}_0}^{\vec{r}} \vec{E} \cdot d\vec{\ell}\,. \tag{4}$$

This potential is really the difference between the potential at $\vec{r}$ and the potential at the reference point and it is what is displayed on a voltmeter when the negative lead is placed on the reference point and the positive lead is placed on the point $\vec{r}$. More generally, the reading displayed on the voltmeter is the amount that the potential at the point of the positive lead is higher than the potential at the point of the negative lead. It gives the amount of work per unit charge that one must do against the electrostatic field to move positive charge from the negative-lead position to the positive-lead position and this equals the amount of work the electrostatic field will do in moving positive charge from the positive-lead position to the negative-lead position.[4]

[3] In any particular problem, the reference point is chosen for convenience. In an electrostatic or electronic circuit, the traditional choice for $\vec{r}_0$ is the negative terminal of a battery or the circuit ground point(s). In the case of a localized set of electrostatical charges, $\vec{r}_0$ is taken as infinitely far away from the charges so $r_0 = \infty$. Note that $\vec{r}_0$ can be a set of points or a surface or a region of points as long as it takes no work to move among those points so "they are all at the same potential."

[4] The voltage across the terminals of flashlight cells is usually 1.5 volts; across lantern batteries, 6 volts; radio batteries, 9 volts; car batteries, 12 volts; and across the terminals in wall outlets, 110 volts or 220 volts (although this is an alternating voltage so the subject benefits from further discussion).

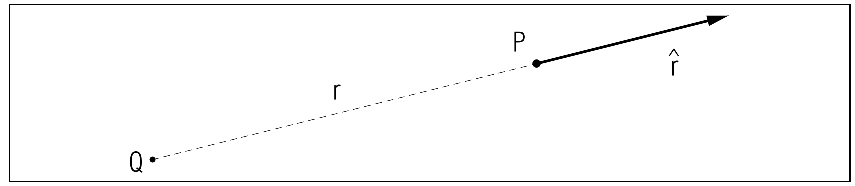


Figure 1. The unit vector $\hat{r}$ at point $\vec{r}$ a distance r from point charge Q.

2c. Case: A Constant Electrostatic Field. Suppose the magnitude of an electrostatic field is constant throughout a region of space, with the value E_0. Suppose also that it has the same direction everywhere in the region. Then for convenience we can orient our coordinate axes so that the electrostatic field is along our x-axis:

$$\vec{E}(\vec{r}) = E_0 \hat{x}\,,$$

where $\hat{x}$ is a unit vector in the direction of increasing x. Then the force on a particle with charge q is:

$$\vec{F} = qE_0 \hat{x}\,,$$

and the particle will have constant acceleration parallel to the x-axis. Let us choose the reference point for the electrostatic potential to be the coordinate origin so the potential is:

$$V(\vec{r}) = V(x) = -\int_0^x E_0 \hat{x} \cdot d\vec{x} = -E_0 \int_0^x dx = -E_0(x - 0) = -E_0 x\,.$$

With the usual orientation of coordinate axes, the force on a positively-charged particle will be to the right so that is the equivalent of the "downhill" direction in the gravitational case. This means that the "uphill" direction must be to the left. The reference point, the "bottom of the hill," is the origin so the "top of the hill", which is in the "uphill" direction from the "bottom", must be to the left of the origin. Since the value of x is negative to the left of the origin, the electrostatic potential is positive there (see Eq. (3)), and that is as it should be for the "top of the hill."

2d. Case: Two Charged Particles. Suppose an electrostatic field is produced by a charged particle Q at the coordinate origin. Then the electrostatic field at the space-point $\vec{r}$ is:

$$\vec{E}(\vec{r}) = k_e \frac{Q}{r^2} \hat{r}\,, \tag{5}$$

where $\hat{r}$ is the unit vector, at the point $\vec{r}$, pointing in the direction of increasing r as shown in Fig. 1. If a particle with charge q is placed at this point it is acted on by an electrostatic force:

$$\vec{F} = k_e \frac{qQ}{r^2} \hat{r} \,,$$

which is just the Coulomb force between two charged particles. For the potential, it is customary and mathematically easiest to take the reference point as any point for which $r = \infty$. Then the potential at r, which is the work-per-unit-charge done against the electrostatic field of Q in bringing a charge from the reference point to radius r, is:[5]

$$V(\vec{r}) = \int_{\infty}^{r} -k_e Q \frac{1}{r^2} dr = k_e \frac{Q}{r} \,.$$

The work-per-unit-charge that the electrostatic field will do to take a charge from r to ∞ is the same:

$$V(\vec{r}) = \int_{r}^{\infty} k_e Q \frac{1}{r^2} dr = k_e \frac{Q}{r} \,.$$

Assuming Q to be positive, the direction of the electrostatic field is away from the origin so that direction, for a positive charge placed "on the hill," is equivalent to "down hill." The "top of the hill" is at the origin and the potential is infinitely positive there. Some hill! If Q is negative, however, note that there is an infintely deep potential hole at the origin!

2e. Using Conservation of Energy. We can use conservation of energy to calculate a particle's change in velocity when it moves from one point to another solely under the influence of electrostatic forces.[6] Fir example, consider the case of two protons that are initially at rest relative to one another and are 10^{-15} m apart.[7] In this configuration, the set of particles has a potential energy equal to 2.3×10^{-13} J and zero kinetic energy. If one of the particles is held fixed while the other is released, the free particle will move away from the fixed one because of the repulsive Coulomb force. Its velocity will increase continuously. How

[5] The integration is: $\int_{\infty}^{r} r^{-2}\, dr = -\, r^{-1}\big|_{\infty}^{r} = -\frac{1}{r} + \frac{1}{\infty} = -\frac{1}{r}$. *Help: [S-4]*

[6] When only conservative forces act on a system, the total energy, kinetic plus potential, is a constant (see MISN-0-21, Sect. 3). As mentioned earlier, electrostatic forces are conservative forces so any particle moving under their sole influence has constant total energy.

[7] Distances of this magnitude are roughly the size of the separation of the particles, neutrons and protons, inside an atomic nucleus.

large a speed will it eventually reach (the proton mass is 1.67×10^{-27} kg)? Note: When the free particle has moved a very large distance away, so that the potential energy of the pair of particles is very close to zero, the total energy of this system is almost all in the form of kinetic energy of the free particle. We find that. for the numbers cited, its eventual speed is 1.66×10^7 m/s, about five percent of the speed of light. *Help: [S-2]*

3. Several Point Charges

3a. Electrostatic Potential. If charges Q_1, Q_2, ..., are located at distances r_{1P}, r_{2P}, ..., respectively, from a point P, then the electrostatic potential at point P is:

$$V = \sum_i k_e \frac{Q_i}{r_{iP}} = k_e \sum_i \frac{Q_i}{r_{iP}} \,.$$

This is the sum of the potentials due to each charge alone. The reason that the contributions from the individual charges simply add is that the potential at any point is defined in terms of the work necessary to bring a test charge from infinity to that point, and work produced by various sources is additive.

3b. Potential Energy. The electrostatic potential energy of a collection of point charges, Q_1, Q_2, ..., is just the sum of the potential energies between all pairs of the charges. Thus the total potential energy of three charges is:

$$E_p = k_e \frac{Q_1 Q_2}{r_{12}} + k_e \frac{Q_1 Q_3}{r_{13}} + k_e \frac{Q_2 Q_3}{r_{23}} \,,$$

where r_{12} is the distance between charges Q_1 and Q_2, etc. If you want to program the rule into a computer, using an arbitrary number of charges, you must write the equation this way:

$$E_p = \frac{1}{2} {\sum_{i,j}}' k_e \frac{Q_i Q_j}{r_{ij}} = \frac{k_e}{2} {\sum_{i,j}}' \frac{Q_i Q_j}{r_{ij}} \,,$$

where the prime on the summation signs mean than you include only terms where $i \neq j$. The quantity r_{ij} is the distance between Q_i and Q_j. The factor of one-half occurs because the summation symbol causes the energy for each pair of particles to be included twice (the term $Q_1 Q_2 / r_{12}$ is the energy for the same pair as is the term $Q_2 Q_1 / r_{21}$). Another way of saying it is that the inter-particle energy for any particular pair is included twice: once for $i > j$ and again for $i < j$.

4. Units

The unit of the potential V(r) is seen from its defining expression, Eq. (4), to be the "newton meter per coulomb," which is also the "joule per coulomb." This unit is called the volt, abbreviated V.

The unit of electrostatic field, the "newton per coulomb", is commonly expressed in its equivalent form, the "volt per meter."

Acknowledgments

The authors would like to thank Professor James Linnemann for useful suggestions. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

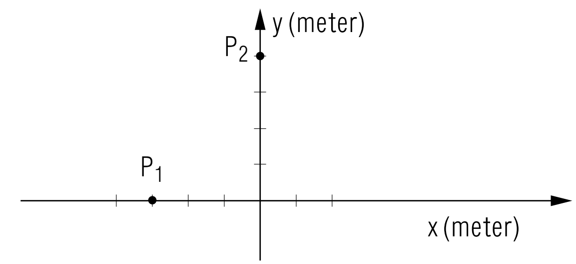
Glossary

- **electrostatic potential**: a scalar field whose value at any point in space is usually defined as the work per unit charge required to bring a charged particle from infinity to that field point. For a point in an electronic circuit, it is the work per unit charge required to bring a charged particle from the circuit's "ground" to the given point (this quantity can be read on a voltmeter).
- **electrostatic potential energy**: the potential energy of a charge or group of charges due to their interaction with an electrostatic field (this quantity is usually computed by multiplying the electrostatic potential at a point in space by electrostatic charge at that point).
- **volt**: the unit of electrostatic potential, abbreviated V; volt = joule per coulomb, usually written $\mathrm{V} = \mathrm{J\,C^{-1}}$.

PROBLEM SUPPLEMENT

Note: Problems 11 and 12 also occur in this module's *Model Exam*.

1.



At point P_1 ($x = -0.300\,\mathrm{m}$, $y = 0.000\,\mathrm{m}$) is located a charge $Q_1 = -4.00 \times 10^{-6}\,\mathrm{C}$. At point P_2 ($x = 0.000\,\mathrm{m}$, $y = 0.400\,\mathrm{m}$) is located a charge $Q_2 = -5.00 \times 10^{-6}\,\mathrm{C}$.

a. Find the potential energy of this system of two charges so arranged.

b. If one of the charges (Q_1) is held fixed and the other (Q_2) is allowed to move an infinite distance away, what now is the potential energy?

c. Where does the difference in potential energy go?

d. If the particle having charge Q_2 also has mass $M = 0.00500\,\mathrm{kg}$, what is its speed when it is an infinite distance from Q_1? *Help: [S-2]*

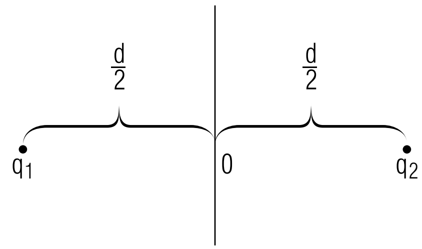
2. Referring to the figure of Problem 1 above, answer the following:

a. Due to the two charges what is the total electric potential at the origin?

b. If a charge $Q_3 = +2.00 \times 10^{-6}\,\mathrm{C}$ is placed at the origin and another charge $Q_4 = +5.00 \times 10^{-6}\,\mathrm{C}$ is placed at $x = +0.300\,\mathrm{m}$, $y = 0.000\,\mathrm{m}$, what is now the total potential energy of this system of four fixed charges? *Help: [S-1]*

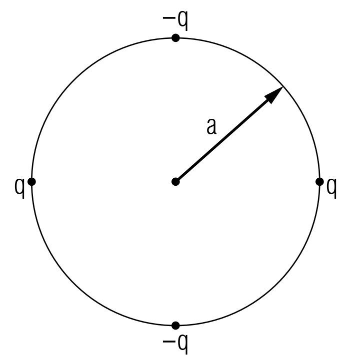
c. To take the particle Q_4 to infinity would you have to do work or would the electric fields do the work for you as they did in part (b) of Problem 1 where the charged particles ended up with kinetic energy?

3. Two charges, $q_1 = +3.0 \times 10^{-6}\,\mathrm{C}$ and $q_2 = +2.0 \times 10^{-6}\,\mathrm{C}$ are fixed in space a distance $d = 5.0\,\mathrm{cm}$ apart, as shown:



What is the electric potential at point O? How much work must be done in order to bring a third charge $q_3 = 2.0 \times 10^{-6}$ C in from infinity to point O?

4. Four charges are placed at intervals of 90° around a circle as shown:



a. Find an expression for the electric potential at the center of the circle.

b. What is the electric potential energy of this charge configuration?

c. Derive an expression for the work required to bring another charge $+q$ in from infinity to the center of the circle.

5. Calculate the electric potential, due to the proton (1.6×10^{-19} C) in a hydrogen atom, at the mean distance of the orbiting electron (5.3×10^{-11} m). What is the potential energy of the electron-proton system, the atom, in joules? What is the potential energy of the atom in units of electron-Volts (eV) where eV$= 1.6 \times 10^{-19}$ J? The energy needed to ionize a hydrogen atom, to remove its electron completely, is known to be 13.6 eV. Explain why this is different from your calculated value for the potential energy.

6. The sodium ions in a salt crystal each have six chlorine ions surrounding them. The chlorine ions can be considered to be lying on the faces of a cube with the sodium ion at its center. If the sodium-chlorine distance is 2.82×10^{-10} m, how much energy is required to completely separate a sodium ion from its neighboring chlorine ions?

Supplementary Problems:

7. Which of the following completions of the statement are correct? (More than one may be correct.) The static electric potential at a point (in a region of space where there are several point charges which contribute to this potential) is:

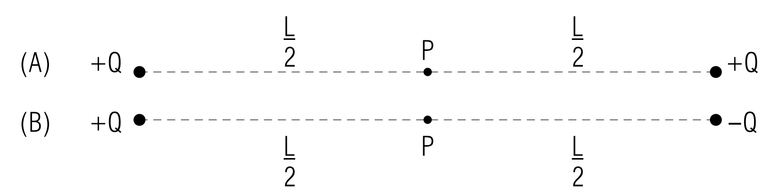
   a. The vector sum of the contributions to this potential from each of the point charges.

   b. The scalar sum of the contributions to this potential from each of the point charges.

   c. The work per unit charge that must be done to bring a positive test charge from an infinite distance to that point in space.

   d. The force per unit charge that would act on a positive test charge placed at the point. *Answer: 12*

8.



Two point charges are separated by a distance L. In case (A) the two are equal charges both in magnitude and sign, both being $+Q$ charges. In case (B) the two charges are of equal magnitude but of opposite sign, $+Q$ and $-Q$. In each of the two cases, find both the electric field intensity, $\vec{E}$, and the electric potential at the point P midway between the two charges.

Case A: $\vec{E}(P) =$ __________ *Answer: 11*; $V(P) =$ __________ *Answer: 7.*

Case B: $\vec{E}(P) =$ __________ *Answer: 13*; $V(P) =$ __________ *Answer: 8.*

9. The units of electric potential are equivalent to which of the composite units below? *Answer: 11*

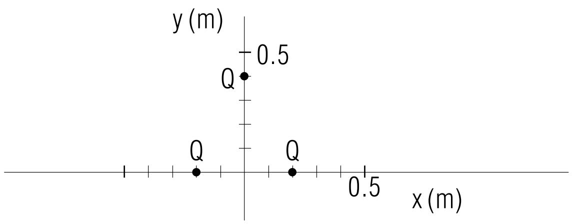
   a. newton per coulomb

   b. joule per coulomb

   c. joule per newton

   d. coulomb per meter

   e. coulomb per joule

10. 

Three point charges, each of charge $Q = +3.00 \times 10^{-5}$ C are arranged at the vertices of an isosceles triangle shown above (one of the charges is at $x = -0.200$ m, $y = 0.000$ m, another is at $x = +0.200$ m, $y = 0.000$ m, and the third is at $x = 0.000$ m, $y = 0.400$ m.)

   a. What is the total potential energy of this system of three charges? *Answer: 14*

   b. How much work would you have to do to bring these three charges together this way (bringing them from infinite relative separations)? *Answer: 9*

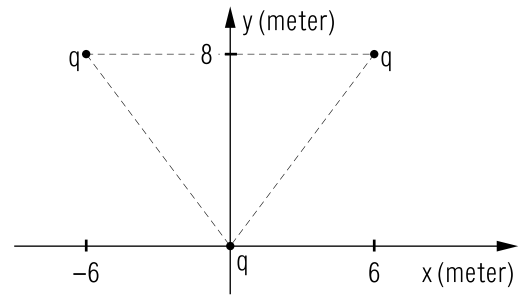
   c. If you released these charges and allowed them to fly apart as a consequence of their mutual repulsion, what total kinetic energy would these have when they got to infinite separation? *Answer: 18*

   d. What is the electric potential at the origin due to this system of three charges arranged in the above triangular configuration? *Answer: 16* What is the potential at $x = +0.3$ m, $y = 0$? *Answer: 10*

   e. How much work would you have to do to bring charge $Q' = +1.0 \times 10^{-6}$ C from $x = +0.30$ m, $y = 0$ to the origin? *Answer: 20*

   f. Due to the triangle of charges, what is the electric field at the origin? *Answer: 13*

   g. What is the force on a charge $Q'' = -1.00 \times 10^{-6}$ C placed at the origin in the presence of the three triangularly arranged charges? *Answer: 15*

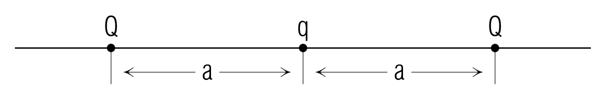
11. 

Three point charges, each of charge $q = 2.0 \times 10^{-6}$ C, are located at (6.0 m, 8.0 m), (−6.0 m, 8.0 m), and at the origin of the above cartesian coordinate system.

   a. Calculate the electric potential at the point (0.0 m, 8.0 m) due to these three point charges. *Answer: 21*

   b. Determine the electric field at the same point. *Answer: 21*

   c. A fourth charge, $Q = 4.0 \times 10^{-6}$ C is placed at the point (0.0 m, 8.0 m) having initially been infinitely distant from all other charge. Calculate the change in the potential energy of the charge Q, due to the above three charges. *Answer: 21*

12. 

$q = +3.0 \times 10^{-6}$ C,

$Q = +5.0 \times 10^{-6}$ C, $a = 2.5$ m

   a. Calculate the potential energy of the above assembly of charges. *Answer: 22*

   b. Calculate the new potential energy of the system if one of the two objects with charge Q is physically interchanged with the object with charge q. *Answer: 22*

c. Use the work-energy principle to determine the work done by the external agent which interchanged the objects in part (b). *Answer: 22*

Brief Answers:

1. a. 0.360 J.

   b. zero.

   c. It goes into kinetic energy of the particle with charge Q_2. Because there are no other forces except the electrical force between Q_1 and Q_2, the total energy, kinetic plus potential, is conserved: $(E_p + E_k)$ at the position shown, equals $(E_p + E_k)$ when Q_2 goes to infinity. At infinity E_p equals zero. When they are as shown in the sketch, $E_K = 0$.

   d. $v = 12.0\,\mathrm{m/s}$. *Help: [S-2]*

2. a. -2.32×10^5 V.

   b. -0.555 joules.

   c. You would need to do work because the potential energy would increase.

3. $$V_0 = k_e \left[\frac{q_1}{d/2} + \frac{q_2}{d/2}\right]$$

$$= (8.99 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}) \left[\frac{(2.0 + 3.0) \times 10^{-6}\,\mathrm{C}}{2.5 \times 10^{-2}\,\mathrm{m}}\right] = 1.8 \times 10^6\,\mathrm{V}$$

$$W = q_3 V_0 = (2.0 \times 10^{-6}\,\mathrm{C})(1.8 \times 10^6\,\mathrm{V}) = 3.6\,\mathrm{J}.$$

4. a. $$V = k_e \left[\frac{q - q + q - q}{a}\right] = 0$$

   b. $$E_p = k_e \left[\frac{q^2}{2a} + \frac{(2q)(-q)}{a\sqrt{2}} + \frac{(2q)(-q)}{a\sqrt{2}} + \frac{(-q)^2}{2a}\right]$$

$$= k_e \frac{q^2}{a}(1 - 2\sqrt{2})$$

   c. $W = qV = 0$

5. $$V = k_e \frac{e}{r} = \frac{(8.99 \times 10^9\,\mathrm{N\,m^2\,C^{-2}})(1.6 \times 10^{-19}\,\mathrm{C})}{5.3 \times 10^{-11}\,\mathrm{m}} = 27\,\mathrm{V}$$

$$E_p = e\,V = (-1.6 \times 10^{-19}\,\mathrm{C})(27\,\mathrm{V}) = -4.3 \times 10^{-18}\,\mathrm{J} = -27\,\mathrm{eV}$$

The electron also has kinetic energy of $(27/2)$ eV.

6. $$W = 6\left(k_e \frac{e^2}{r}\right) = (6)(8.99 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}) \left(\frac{(1.6 \times 10^{-19}\,\mathrm{C})^2}{2.82 \times 10^{-10}\,\mathrm{m}}\right) = 4.9 \times 10^{-18}\,\mathrm{J}$$

7. $4 k_e Q/L$

8. Zero

9. 56.5 J

10. 3.78×10^6 V

11. (b)

12. (b) and (c) are correct

13. $-(1.69 \times 10^6\,\mathrm{N/C})\hat{y}$

14. 56.5 J

15. $+(1.69\,\mathrm{N})\hat{y}$

16. 3.38×10^6 V

17. Zero

18. 56.5 J

19. $8 k_e Q/L^2$ directed to the right

20. $Q'(V_f - V_i) = -0.405$ J [you wouldn't have to do the work, the field would do it; work done by field= $Q'(V_i - V_f)$].

21. a. $V = 8.2 \times 10^3$ V

    b. $\vec{E} = (2.8 \times 10^2\,\mathrm{V/m})\hat{y}$ or $(2.8 \times 10^2\,\mathrm{N/C})\hat{y}$

c. $\Delta E_p = 0.033\,\mathrm{J}$

22. a. $E_p(\text{initial}) = 0.15\,\mathrm{J}$

b. $E_p(\text{final}) = 0.17\,\mathrm{J}$

c. $W_{\text{external}} = +0.018\,\mathrm{J}$

SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from PS, Problem 2)*

The potential energy is the work you must do in assembling the system of charges, starting with the charges infinitely distant from one another [i.e. $E_p(\text{initial}) = 0$]. The work done in assembling the pair Q_1 and Q_2 was calculated in Problem 1a. The work done in bringing Q_3 from infinity to the origin can be calculated from knowing the potential at infinity (zero) and at the origin due to Q_1 and Q_2 (Problem 2a). To find the potential energy change when Q_4 is brought to the point $(0.3\,\mathrm{m}, 0)$, calculate the potential at this point due to Q_1, Q_2 and Q_3 and apply Eq. (5). The result is:

$$\begin{aligned} E_p(\text{total}) &= E_p(\text{initial}) + \Delta E_p(Q_1 + Q_2) + \Delta E_p(Q_3) + \Delta E_p(Q_4) \\ &= 0 + 0.36\,\mathrm{J} + (2 \times 10^{-6}\,\mathrm{C})(-2.3 \times 10^5\,\mathrm{V}) + Q_4 V_{123}(0.3\,\mathrm{m}, 0) \end{aligned}$$

where V_{123}(0.3 m,0) is the potential at (x = 0.3 m,y = 0) due to charges Q_1, Q_2, and Q_3. Alternatively you could use the expression:

$$E_p(\text{total}) = k_e \sum \frac{Q_i Q_j}{r_{ij}}$$

where r_{ij} is the distance between the pair of charges Q_i and Q_j, and the summation is over all charge pairs. Neither count a charge pair twice nor calculate the potential energy of interaction of a charge with itself!

S-2 *(from TX, 2c and PS, Problem 1d)*

From mechanics, recall the relationship between a particle's kinetic energy, velocity and mass. In this problem we know the particle's kinetic energy and mass, so we can solve for its velocity.

S-3 *(from PS, Problem 5)*

Recall that the electron charge is just the negative of the proton charge. You can easily get the answer in joules from the answer given.

S-4 *(from TX, 2b, Eq. (2))*

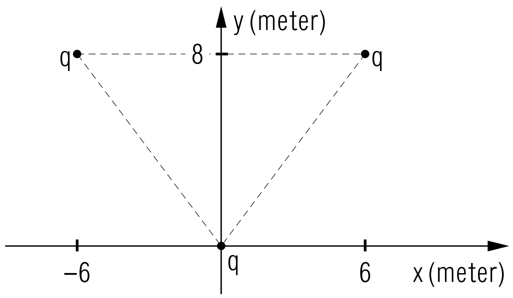
$$\int x^{-n}\,dx = \frac{1}{-n+1} x^{-n+1}$$

MODEL EXAM

$$k_e = 8.99 \times 10^9\,\mathrm{N\,m^2/\,C^2}$$

1. See Output Skill K1 in this module's *ID Sheet*.

2.



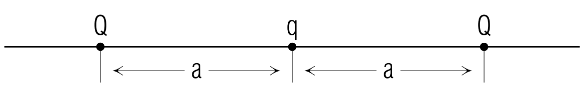
Three point charges, each of charge $q = 2.0 \times 10^{-6}\,\mathrm{C}$, are located at $(6,8)$, $(-6,8)$, and at the origin of the above cartesian coordinate system.

a. Calculate the electric potential at the point (0,8) due to these three point charges.

b. Determine the electric field at the same point.

c. A fourth charge, $Q = 4 \times 10^{-6}\,\mathrm{C}$ is placed at the point $(0,8)$ having initially been infinitely distant from all other charge. Calculate the change in the potential energy of the charge Q, due to the above three charges.

3.



$q = +3.0 \times 10^{-6}\,\mathrm{C}$,

$Q = +5.0 \times 10^{-6}\,\mathrm{C}$, $a = 2.5\,\mathrm{m}$

a. Calculate the potential energy of the above assembly of charges.

b. Calculate the new potential energy of the system if one of the Q's is interchanged (exchanged, switched) with q.

c. Use the work-energy principle to determine the work done by the external agent which interchanged the charges in part (b).

Brief Answers:

1. See this module's *text*.
2. See Problem 11 in this module's *Problem Supplement*.
3. See Problem 12 in this module's *Problem Supplement*.