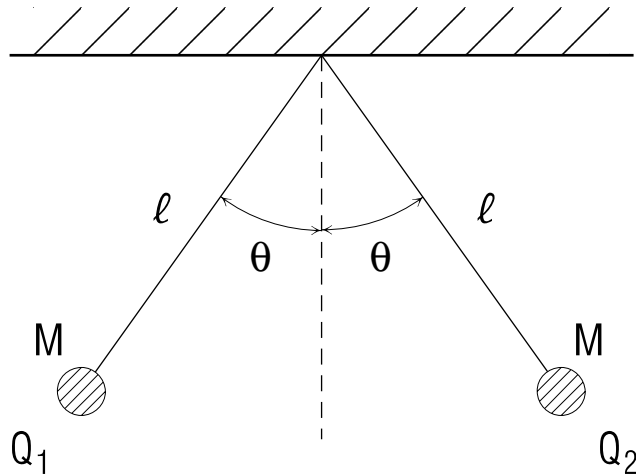


COULOMB'S LAW



COULOMB'S LAW

by
J. Kovacs

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Title: **Coulomb's Law**

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Input Skills:

1. Vocabulary: electric charge, electric force (MISN-0-121).
2. State Newton's second and third laws (MISN-0-16).
3. Express an arbitrary vector in terms of the unit vectors associated with a fixed coordinate system (MISN-0-2).
4. Add two or more vectors and determine the magnitude and direction of their resultant (MISN-0-2).

Output Skills (Knowledge):

- K1. Vocabulary: charge, coulomb, Coulomb force, electrostatic (interaction), electrostatic force constant (k_e).
- K2. State Coulomb's law and identify each of the quantities in the expression, labeling each with its appropriate dimensions. Also draw a sketch showing two charges and labeled with the quantities in Coulomb's law, showing the direction of the forces involved in relation to the signs and positions of the charges.

Output Skills (Problem Solving):

- S1. Calculate the magnitude and direction of the resultant force exerted on a given point charge by a given spatial arrangement of other point charges.

Post-Options:

1. "Point Charge: Field and Force" (MISN-0-115).
2. "Electrostatic Potential Due to Discrete Charges" (MISN-0-116).

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by
J. Kovacs

1. Introduction

One of the fundamental interactions in nature is the electromagnetic interaction. It is the interaction that is most directly responsible for all of the chemical and biological phenomena that you observe: the colors of objects, the solidness of a brick wall, the vapor pressure of mercury, the combustibility of gasoline, etc. Contrasted with this, your weight on the surface of the earth is a consequence of the gravitational interaction, while the energy output of the sun and the stars is a consequence of the nuclear interaction. The basic property associated with the electromagnetic interaction is the electric charge that substances carry. This can be attributed to the charge carried by the microscopic constituents of matter. The basic consequence of the existence of these charges is that a charged particle exerts a force on another charged particle. The precise nature of this force is expressed in Coulomb's law.

2. Force between Two Point Charges

2a. Static Two-Body Gravitational Force. Two stationary masses exert an attractive force on one another that is proportional to their masses and inversely proportional to the square of their separation r :¹

$$|\vec{F}_{\text{grav}}| = G \frac{m_1 m_2}{r^2}. \quad (1)$$

This is a mutually attractive force: each mass attracts the other with this same force, in agreement with Newton's third law. Although the magnitudes of the forces on the two are equal, their accelerations will not be equal if their masses are not equal. For example, the sun and the earth each exert the same force on the other but the earth's acceleration toward the sun is 10^5 times the sun's acceleration toward the earth.²

2b. Two-Body Electrostatic Force. In addition to the gravitational force (and apparently independently of any gravitational force), two particles may exert a force on each other whose origin is attributed to another

¹See "Newton's Law of Universal Gravitation" (MISN-0-101).

²Recall Newton's second law.

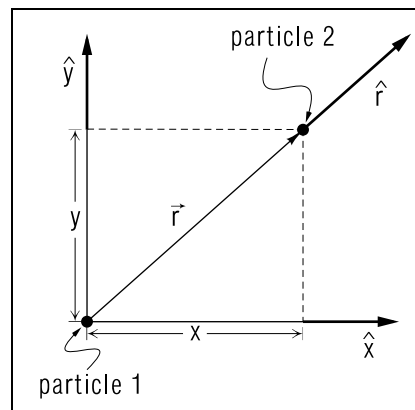


Figure 1. Distances and directions for illustrating the Coulomb force of Particle 1 on Particle 2.

basic property of the particles, electric charge. That is, in addition to the mass, each particle has an electric charge, the particles will exert a force on one another that is a consequence of the existence of their charges. This force is called the electrostatic force.³ Unlike the gravitational force, which can only be attractive, the electrostatic force can be either repulsive or attractive. Furthermore, for the masses and charges of the elementary particles of nature such as the electrons and protons, the electrostatic force is many orders of magnitude larger than the gravitational force between the same two particles.

2c. The Electrostatic Force (Qualitative). Qualitatively, the force that two charged particles exert on each other via the electrostatic interaction can be described as: (a) always along the line joining them, (b) either repulsive or attractive depending upon whether the charges have the same sign or have opposite signs, and (c) dependent upon the separation of the particles, with the magnitude of the force decreasing with increasing separation between them. In fact, given the particles' charges and their locations, one can calculate the force each exerts on the other. The mathematical expression that yields the force is called Coulomb's law. It is known to be valid over the remarkable range of separation distances from as small as 10^{-15} meters to as large as 10^6 meters.⁴

³The "static" stresses the fact that this force exists even if the particles are stationary relative to the observer. This is to distinguish it from the magnetic force that arises when charged particles move relative to the observer.

⁴This is not to say that the result is incorrect at distances larger than a million meters; that number is merely the upper limit to which experiment has verified this inverse-square-law force.

2d. Coulomb's Law. The simplest expression that demonstrates how the electrostatic force depends upon the separation between the two charged particles can be written down referring to Fig. 1. The magnitude of the electric force exerted on either particle by the other is:

$$|\vec{F}_e| = \frac{K}{r^2}. \quad (2)$$

The coefficient K is independent of the particle separation, depending only on a basic intrinsic property of the two particles themselves, their electrical charges.⁵ The coefficient K can be both positive and negative, unlike the gravitational counterpart which is always negative, indicating both repulsion and attraction as possible between the two charged particles. Because the electrostatic force can be demonstrated to be proportional to the charge on each of the particles, the product of the individual charges appears in K . Taking this into account, Eq. (1) may be written for the force exerted on charged particle 2 by charged particle 1:

$$\vec{F}_{e(\text{on particle 2})} = k_e \frac{Q_1 Q_2}{r^2} \hat{r}, \quad (3)$$

where Q_1 and Q_2 are the charges of particles 1 and 2, respectively, and \hat{r} is the unit vector pointing along the radius vector from particle 1 to particle 2 (see Fig. 1). The unit of charge that enters in this expression, when F and r are in MKS units, is the ‘‘coulomb.’’ The constant quantity k_e is called ‘‘the electric force constant.’’ Its value is *defined* to be:

$$k_e = 8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}. \quad (4)$$

The newton and meter are defined elsewhere,⁶ while the coulomb (symbol ‘‘C’’) is defined in terms of electrical current and time. Note that k_e is the force in newtons that two identical one-coulomb charges would exert on one another if they were separated by a distance of one meter, just as G is the gravitational force, in newtons, that two one-kilogram masses would exert on each other when separated by one meter.

⁵Similarly, the gravitational force between two particles depends upon another intrinsic property of the particles, their masses. The expression for the gravitational force between two particles (see MISN-0-101) is similar to Eq. (1), both of them decreasing as the inverse square of the inter-particle separation. The distance-independent coefficients, however, are vastly different. As an illustration of this, the electrostatic force between two electrons for a given separation is exactly the same as the electrostatic force between two protons. However, the gravitational force between protons is more than 3 million times the gravitational force between two electrons for the same separation.

⁶See the Tables at the end of this book.

2e. Basic Electric Charges. Normal atoms and molecules consist of electrons, protons, and neutrons, and charge is strictly additive when these particles combine to form composite particles or collections of particles. Electrons, protons, and neutrons have these amounts of charge:⁷

$$Q_e = -(1.60217733 \pm 0.00000049) \times 10^{-19} \text{ C},$$

$$Q_p = +(1.60217733 \pm 0.00000049) \times 10^{-19} \text{ C},$$

$$Q_n = (-0.4 \pm 1.1) \times 10^{-21} |Q_e|.$$

We will generally require only three digits of accuracy so we take the charge on the neutron as zero and the charges on the proton and the electron as equal but opposite in sign. For notation, we use $Q_e = -Q_p = -e$ where:

$$e = 1.602 \times 10^{-19} \text{ C}. \quad (5)$$

All charges that are ever observed are positive or negative integer multiples of the charge on the electron (or proton).⁸ Obviously, it takes a large number of electrons and/or protons to make a coulomb or, for that matter, any of the normally observed amounts of charge that are dealt with in standard electronic apparatus.

3. Several Point Charges

3a. Choose a Convenient Coordinate System. When there are more than two charges, the introduction of a suitable coordinate system with a corresponding set of unit vectors greatly simplifies the bookkeeping involved in determining the force on any one of the particles. An example with three charges is shown in Fig. 2.

3b. Apply Coulomb's Law to All Charge Pairs. Coulomb's law still describes the electrostatic interaction between all pairs of point charges in the system. There are two contributions to the force on the charge Q_1 due to charges Q_2 and Q_3 in Fig. 2: one that would be exerted on Q_1 if only the pair Q_1 and Q_2 were present and the other that would be exerted on Q_1 if only the pair Q_1 and Q_3 were present. These forces are:

$$\text{Due to } Q_3 : \vec{F}_3 = k_e \frac{Q_1 Q_3}{R_{13}^2} \hat{y},$$

⁷Actually, $|Q_p + Q_e| < 1.0 \times 10^{-21} e$.

⁸The particles called ‘‘quarks’’ constitute the only exception: they are the constituents of protons and neutrons and their charges are one-third and two-thirds the electronic charge e .

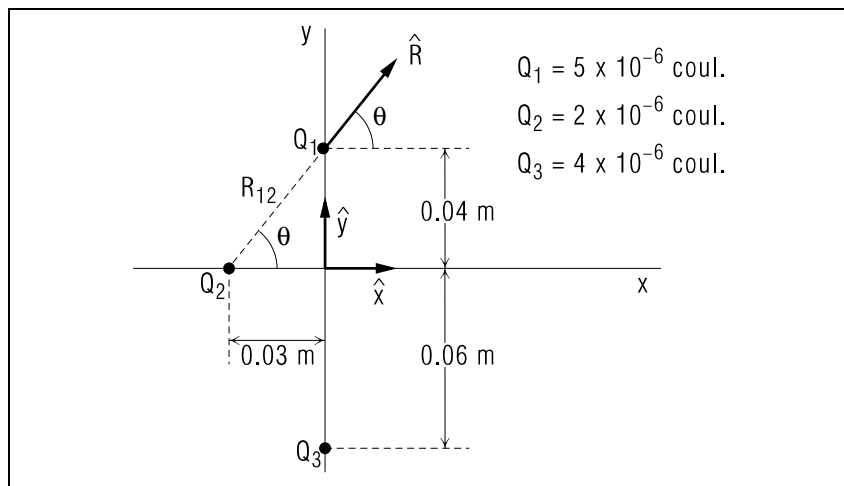


Figure 2. A convenient coordinate system for three point charges.

where R_{13} is the distance between the charges Q_1 and Q_3 and \hat{y} is a unit vector directed along the positive y -axis. (The force that Q_1 exerts on Q_3 is the negative of the above.)

$$\text{Due to } Q_2 : \vec{F}_2 = k_e \frac{Q_1 Q_2}{R_{12}^2} \hat{R}.$$

Where \hat{R} is a unit vector ($|\hat{R}| = 1$) pointing along the line from Q_2 to Q_1 .

3c. Total Force is a Vector Sum. The resultant force on Q_1 in Fig. 2 is the vector sum of the forces from the other two charges:

$$\vec{F} = \vec{F}_2 + \vec{F}_3 = k_e \left(\frac{Q_1 Q_3}{R_{13}^2} \hat{y} + \frac{Q_1 Q_2}{R_{12}^2} \hat{R} \right). \quad (6)$$

To combine these two vectors it is necessary to write unit vector \hat{R} in terms of its x and y -components:

$$\hat{R} = (\cos \theta) \hat{x} + (\sin \theta) \hat{y}, \quad \text{Help: [S-2]} \quad (7)$$

with θ as defined in Fig. 2. Then the net force is:

$$\begin{aligned} \vec{F} &= \vec{F}_2 + \vec{F}_3 \\ &= k_e \left[\left(\frac{Q_1 Q_3}{R_{13}^2} + \frac{Q_1 Q_2 \sin \theta}{R_{12}^2} \right) \hat{y} + \frac{Q_1 Q_2 \cos \theta}{R_{12}^2} \hat{x} \right]. \end{aligned} \quad (8)$$

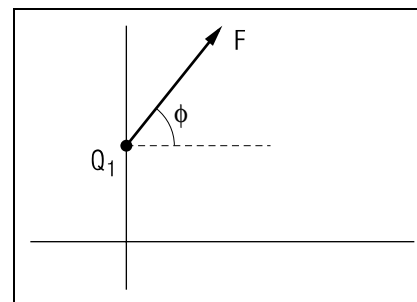


Figure 3. The resultant force on charge Q_1 due to the arrangement of charges shown in Fig. 2.

3d. Numerical Example. Numerically, with $R_{13} = 0.100$ m, $R_{12} = 0.0500$ m, $\sin \theta = 0.800$, and $\cos \theta = 0.600$, the net force on Q_1 in Fig. 2 can be determined to be

$$\vec{F} = (46.8 \text{ N}) \hat{y} + (21.6 \text{ N}) \hat{x}. \quad (9)$$

The magnitude of this force is

$$|\vec{F}| = 51.5 \text{ newtons}. \quad (10)$$

The direction of the force makes an angle $\phi = 65.2^\circ$ with the positive x -axis as shown in Fig. 3. This is a good example to check for yourself.

Acknowledgments

Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

Glossary

- **charge:** the basic property of a piece of matter that defines the strength of its interaction with other matter through electromagnetic forces.
- **coulomb:** the MKS unit of charge, abbreviated “C,” defined as the amount of charge that passes a point in one second due to a current of one ampere (“ampere” and “second” are basic SI units).
- **Coulomb force:** an interaction between two charged particles that obeys Coulomb’s law, i.e. that depends on the product of the charges and inversely on the square of the particles’ separation.

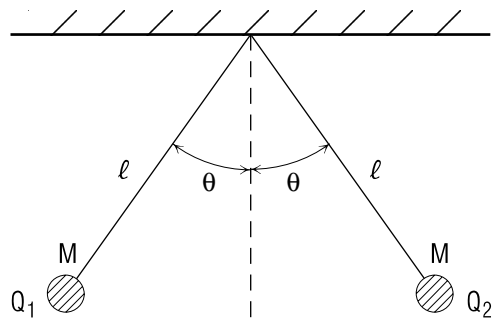
- **electrostatic (interaction):** an electromagnetic interaction that is independent of the motion of charged particles.
- **electrostatic constant:** also called the electrostatic force constant, written k_e . It is the intrinsic strength of the interaction between charged particles and as such appears in Coulomb's law and other electric and magnetic expressions. Its value is *defined* to be: $k_e \equiv c^2 10^{-7} \text{ N s}^2 \text{ C}^{-2} = 8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$. Here c is the speed of light.

PROBLEM SUPPLEMENT

Note: Problem 6 also occurs in this module's *Model Exam*.

- For the system of three charged particles shown in Fig. 2, derive the expression that gives the resultant force on charge Q_1 due to charges Q_2 and Q_3 :
 - Express the resultant in terms of the unit vectors \hat{x} and \hat{y} associated with the coordinate system shown in Fig. 2, getting Eq. (8).
 - Given the numerical values shown in Fig. 2, evaluate the values of the x and y -components of this resultant force, as shown in Eq. (9).
 - Putting in the dimensions of all of the quantities that enter into the equation used in part (b), above, verify that the correct dimensions of the force components are the "newton."
 - Determine the magnitude and direction of this resultant force on charge Q_1 , getting Eq. (10) and Fig. 3.
- Write down the expression for Coulomb's law. Put in the units for each one of the factors in the expression (including the electrostatic constant k_e) and verify that you wind up with a force unit. *Answer: 11*
- A charge $Q_1 = -3.0 \times 10^{-6} \text{ C}$ is located at the origin of a cartesian coordinate system while a charge $Q_2 = +4.0 \times 10^{-6} \text{ C}$ is located at $x = +0.30$ meters, $y = 0$, $z = 0$.
 - Calculate the force on Q_1 due to Q_2 . *Answer: 2*
 - Calculate the force on Q_2 due to Q_1 . *Answer: 8*

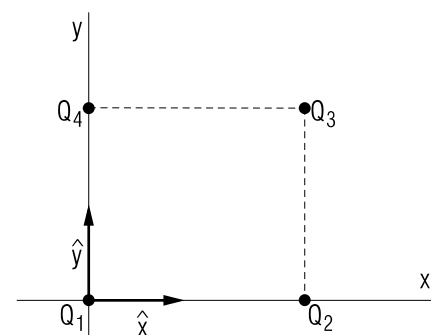
4.



Two balls each of mass $M = 0.0500$ kilograms are charged and attached to two strings of length $\ell = 0.600$ meters hanging from the same point. When the system is in equilibrium each of the balls makes an angle of 30.0° with the vertical.

- Draw a one-body force diagram for the ball carrying charge Q_2 and identify the object that exerts each of those forces on this ball. *Answer: 14*
- Do the same for the other ball. *Answer: 1*
- Set up a coordinate system at the location of the ball carrying charge Q_2 with \hat{x} pointing horizontally to the right, \hat{y} vertically upward. Express the three forces acting on this ball in terms of these unit vectors and the symbols in the sketch (above). *Answer: 5* What is the resultant (symbolic) force on the ball, based on its acceleration? *Answer: 2*
- The values of ℓ , M , and θ on the left side of the sketch (above) are the same as those on the right side of the sketch. Does this mean that the value of the charge on the left side of the sketch is the same as that on the right side (i.e., does $Q_1 = Q_2$)? *Answer: 16*
- If $Q_1 = 4.00 \times 10^{-6}$ C, what is the value of Q_2 ? *Answer: 6*

5.



At the corners of a square of side length $d = 0.030$ m are located four charges as shown in the diagram above. Q_1 is located at the origin of the coordinates, Q_4 is on the y -axis and Q_2 on the x -axis.

$$Q_1 = -3.0 \times 10^{-6} \text{ C}$$

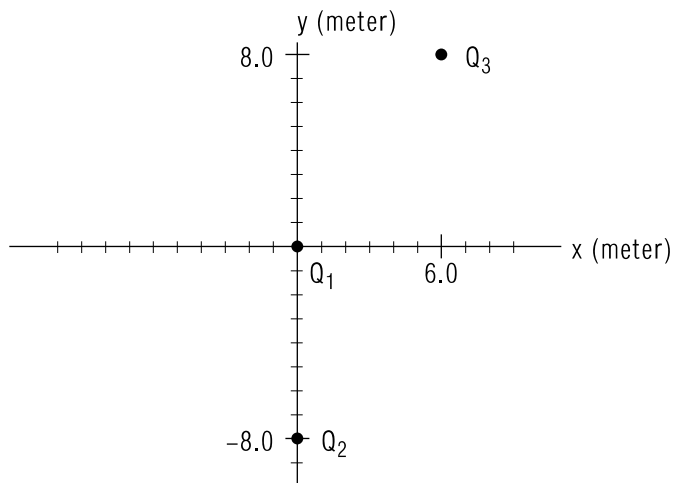
$$Q_2 = -9.0 \times 10^{-6} \text{ C}$$

$$Q_3 = +2.0 \times 10^{-6} \text{ C}$$

$$Q_4 = +1.0 \times 10^{-6} \text{ C}$$

- What force does the charge Q_1 exert on Q_3 ? *Answer: 13* What force does Q_3 exert on Q_1 ? *Answer: 3*
- What force does Q_4 exert on Q_3 ? *Answer: 10*
- What force does Q_2 exert on Q_3 ? *Answer: 4*
- What is the resultant force on Q_3 ? *Answer: 9*
- Calculate the direction of this net force measured relative to the positive x -axis. *Answer: 7*

6. Three point charges are located at positions as indicated in this diagram:

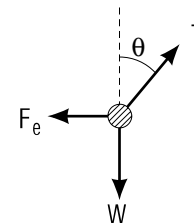


- $Q_1 = +5.0 \times 10^{-4}$ C, located at the origin
 $Q_2 = -2.0 \times 10^{-4}$ C, located at (0, -8.0 m)
 $Q_3 = +8.0 \times 10^{-4}$ C, located at (6.0 m, 8.0 m)

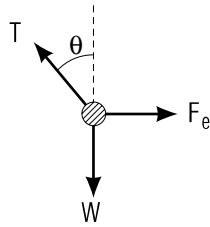
Calculate the magnitude and direction of the resultant force on charge Q_3 . *Answer: 15*

Brief Answers:

1. T , F_e , W same as in answer N , except that F_e in this case is the force on Q_1 due to Q_2 .



2. $(1.2 \text{ N}) \hat{x}$
 3. $(21 \text{ N}) \hat{x} + (21 \text{ N}) \hat{y}$
 4. $-1.80 \times 10^2 \text{ N } \hat{y}$
 5. $\vec{F}_e = k_e \frac{Q_1 Q_2}{4l^2 \sin^2 \theta} \hat{x}$; $W = -Mg \hat{y}$; $\vec{T} = (T \cos \theta) \hat{y} - (T \sin \theta) \hat{x}$
 6. $2.83 \times 10^{-6} \text{ C}$ *Help: [S-1]*
 7. 90° clockwise from the positive x -direction
 8. $-1.2 \text{ N } \hat{x}$
 9. $-1 \text{ N } \hat{x} - 2.0 \times 10^2 \text{ N } \hat{y}$
 10. $2.0 \times 10^1 \text{ N } \hat{x}$
 11. See the module text
 12. It is not moving, so $\vec{a} = 0$ so $\vec{F} = \vec{F}_e + \vec{T} + \vec{W} = 0$
 13. $(-21 \text{ N}) \hat{x} - (21 \text{ N}) \hat{y}$
 14. T = contact force exerted by the string
 F_e = non-contact electrostatic force exerted by charge Q_1
 W = non-contact gravitational force exerted by the earth



15. $F = 31$ N, away from the origin at an angle that is 51° from the positive x -axis and 39° from the positive y -axis.
16. No, but Coulomb's law produces a force of Q_1 on Q_2 that is equal and opposite to the force of Q_2 on Q_1 , regardless of the relative sizes of Q_1 and Q_2 . This is in agreement with Newton's third law.

SPECIAL ASSISTANCE SUPPLEMENT

S-1

(from PS, Problem 4d)

$$Q_2 = \frac{(.05 \text{ kg})(9.8 \text{ m/s}^2)(4)(.6 \text{ m})^2(\sin 30^\circ)^2(\tan 30^\circ)}{(4.00 \times 10^{-6} \text{ C})(8.99 \times 10^9 \text{ N m}^2/\text{C}^2)} = 2.83 \times 10^{-6} \text{ C}$$

Note: distance between charges = $(2)(0.6 \text{ m})(\sin 30^\circ) = 0.6 \text{ m}$. An alternative: note the equilateral triangle formed by the two strings and an imaginary line connecting the balls: all three angles are equal so all three sides are equal. Anyway, sketch the situation roughly to scale and see that the value of the distance is correct.

S-2

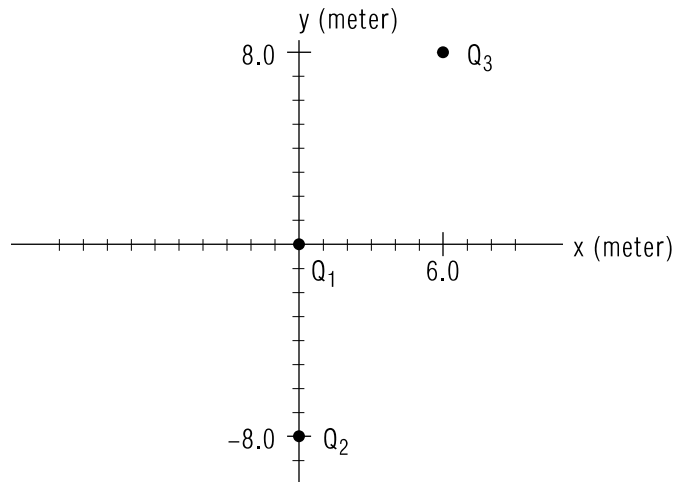
(from TX, 3c)

If you have trouble with this equation, review MISN-0-1 where such expressions are discussed in great detail. You can always check such an expression by taking: (i) $\hat{R} \cdot \hat{R}$ to see that it is indeed of unit length; (ii) $\hat{R} \cdot \hat{x}$ to see that the x -component is correct; and (iii) $\hat{R} \cdot \hat{y}$ to see that the y -component is correct.

MODEL EXAM

$$k_e = 8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

1. See Output Skills K1-K2 in this module's *ID Sheet*.
2. Three point charges are located at positions as indicated in this diagram:



- $Q_1 = +5.0 \times 10^{-4} \text{ C}$, located at the origin
 $Q_2 = -2.0 \times 10^{-4} \text{ C}$, located at $(0, -8.0 \text{ m})$
 $Q_3 = +8.0 \times 10^{-4} \text{ C}$, located at $(6.0 \text{ m}, 8.0 \text{ m})$

Calculate the magnitude and direction of the resultant force on charge Q_3 .

Brief Answers:

1. See this module's *text*.
2. See this Problem 6 in this module's *Problem Supplement*.