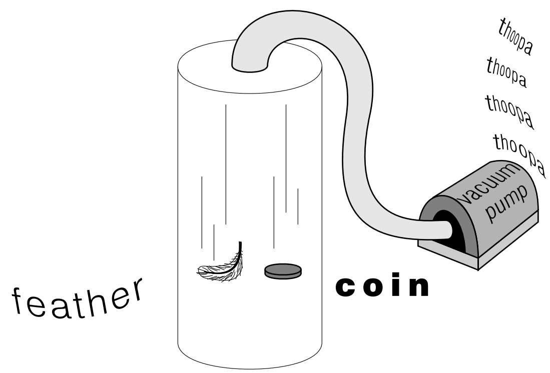
MISN-0-110

THE EQUIVALENCE PRINCIPLE: AN INTRODUCTION TO RELATIVISTIC GRAVITATION



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

THE EQUIVALENCE PRINCIPLE: AN INTRODUCTION TO RELATIVISTIC GRAVITATION

by

Peter Noerdlinger

Title: **The Equivalence Principle: An Introduction to Relativistic Gravitation**

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Input Skills:

1. Vocabulary: Archimede's Principle (MISN-0-48), torsion pendulum (MISN-0-100), centripetal and Coriolis forces MISN-0-18).
2. Solve problems involving relative linear motion (MISN-0-11).

Output Skills (Knowledge):

K1. Vocabulary: gravitational mass, inertial mass, active and passive gravitational mass, (generalized) tidal force.

K2. Explain how Galileo's experiment and Newton's second law establish the proportionality of gravitational force to mass.

K3. Outline the methods, actual and idealized, used to measure gravitational and inertial mass.

K4. Give three examples where inertial forces come into play on you.

K5. State: why Einstein suspected that inertial and gravitational forces were essentially the same; and to what level of precision they are known to be equivalent.

K6. State one way you could tell that the Earth's pull was present if you were an astronaut orbiting the earth in a sealed capsule of finite size, and another way if you had a window.

K7. State the objective of the Dicke-Eötvös experiment. Describe what is looked for in the behavior of the torsional penelulum and why.

K8. Design a simple experiment that could be used to measure the local acceleration of gravity, $\vec{g}$. Show that the velocity of the article being used is immaterial to the measurement.

K9. Compare Einstein's Equivalence Principle (ignoring any limitations) with Newton's explanation, assuming the earth's gravity field is present.

K10. State whether you believe Newton's third law is absolute and justify your belief.

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by

Peter Noerdlinger

1. Introduction: Galileo and Newton

1a. Galileo's Universal-Acceleration Discovery. Galileo's most far-reaching discovery was that insofar as air resistance can be neglected, all objects released at rest from the same point fall with like acceleration (and hence motion) under the influence of gravity. This is commonly illustrated by the experiment of the coin and feather in a tube that can be evacuated. Galileo's result has been confirmed to our best experimental accuracy[1] (in some cases reaching 11 significant figures) for all kinds of materials and objects, ranging from aluminum and gold to chewing gum, planets in their orbits, and free neutrons (uncharged "elementary" constituents of most nuclei).[2]

1b. Newton's Explanation. Naturally, Galileo's principle of "equal falling" raises the question: Why? The two answers that have been found satisfactory in different contexts were provided, respectively, by

[1] See "How Exact an Equivalence?" which is Sect. 7.

[2] See "Fundamental Forces and Elementary Particle Classifications" (MISN-0-225).

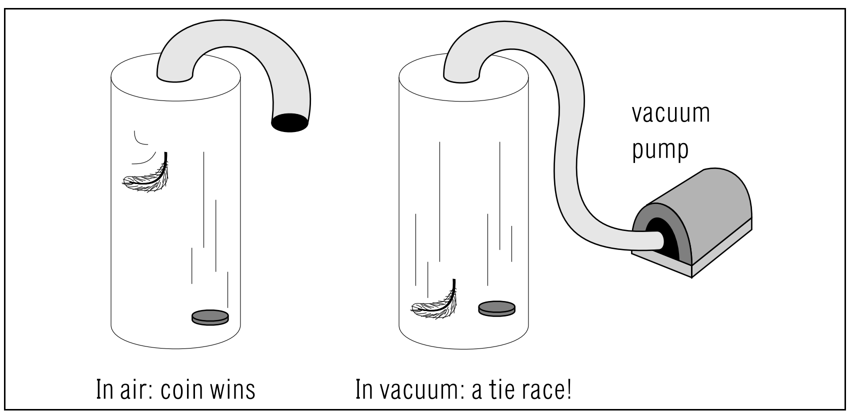

Figure 1. .

Isaac Newton and Albert Einstein. Newton's answer is simplest to understand and we can derive it at once, using his second law of motion. If a BB shot of mass 0.2 g and a bowling ball of mass 2 kg (10^4 times larger) are to fall identically under the law

$$\vec{F} = m\,\vec{a}, \tag{1}$$

then the force $\vec{F}$ on the ball must be 10^4 times that on the BB. In other words, we must assume that the force of gravity is proportional to the very mass m that we used in Eq. (1). So we write

$$\vec{F} = m\,\vec{g} \tag{2}$$

where $\vec{g}$ is a constant (for a given location) independent of mass or composition. For the surface of the Earth, $\vec{g}$ is found to be within 1% of $9.80\,\mathrm{m/s^2}$, directed down at all locations. So Newton decided that the "equal falling" in vacuum can be said to be due to what we shall call *Newton's Explanation*:

Newton's Explanation	At any particular point in space and time, the gravitational force on each member of a collection of objects will be proportional to its Newton's-Second-Law mass.

We call an object's mass measured using acceleration, not gravity, its *inertial* mass; and the (presumably equal) mass using gravity, not acceleration, its *gravitational* mass.

2. Mass Measurement

2a. Measuring Inertial Mass. There can be content to Newton's Explanation only if we can define the inertial mass independently of gravity! In fact that is about the first thing we learn in physics.[3] To measure inertial mass, we could set the unknown mass on a disc on a frictionless surface (such as an air table), pull it with a constant force, and measure its acceleration (Fig. 2). Then we put standard masses on the puck until the same force produces the same acceleration. The total mass is then equal to m. Or, we could take the object out in free space, attach it to a propulsion device of known thrust, and measure the resulting acceleration. Then we again use standard masses until we find a combination where the same F produces the same a in both cases. These dynamic systems measure inertial mass, and as such, have no dependence on gravity.

[3] See "Particle Dynamics - The Laws of Motion" (MISN-0-14).

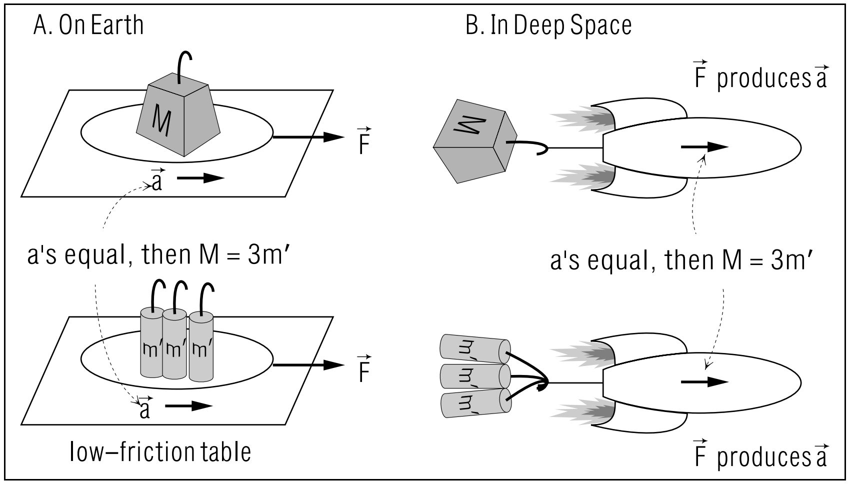

Figure 2. .

2b. Measuring Gravitational Mass. The gravitational mass is presumably equal to the inertial mass through Newton's Explanation, but we also want to be able to define it independently so that we can test the explanation's accuracy. In practice, we usually use instead a spring scale or balance, as in Fig. 3.

With a spring, we can measure the extension caused by the object, and then find the number of standard masses that produces the same extension. Or, we could balance the mass with the proper number of standard masses. Since the gravitational force is then the same on both sides, the mass is the same on both sides. These static systems measure gravitational mass, and have total dependence on gravity.

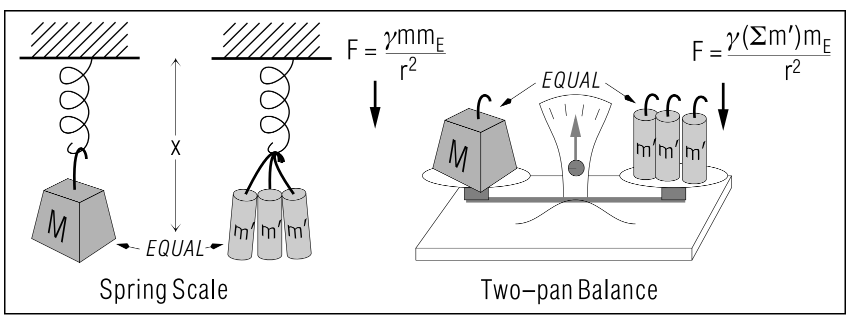

Figure 3. .

3. Einstein's Explanation

3a. Einstein's Approach. If Newton settled the mass equivalence so well, why did Einstein go on to a more sophisticated view? Can't we be satisfied with Newton's explanation? Einstein looked for deeper causes. Why should the force of gravity be proportional to (inertial) mass? Electrical forces are not proportional to mass: a comb rubbed through our hair will pick up bits of paper, but not a whole book! If we pull on a rolling object (or on an object in outer space) with a cable, the force exerted is proportional to both mass and acceleration, not just to mass! Gravity is, in a sense[4] the only force proportional to mass alone. Why? It could be just a "fact of life": we are never sure in advance if our "why's" have answers. But Einstein had, besides a deep vision, some further guideposts. One was the special theory of relativity, which we'll steer around here. But another is easy to understand and quite practical. It can be phrased in terms of "inertial" forces (also called "fictitious forces" by some physicists); these are forces that seem like gravity (and are measured frequently in "g's") which are present in an accelerated reference frame, such as a rapidly braking car. Einstein's answer was to say that such inertial forces and gravitational forces are indistinguishable.

3b. The Reality of Inertial Forces. Inertial forces can seem very real, as you find out when you hit the windshield of the car, or if you're in a centrifuge. Some carnivals have centrifuges big enough to hold a person[5] and if you're in one you feel a "force" like gravity pulling you toward the rim. The centripetal force is the force, as measured by the carnival operator in his non-rotating inertial frame of reference, that keeps you in circular motion.[6] The centrifugal force ("inertial," "fictitious," but let's drop that baloney) is the one that you perceive as forcing you out toward the rim.[7] The Coriolis force is an additional inertial force experienced by objects which are moving in a rotating reference frame. It is sufficiently real that cannons were missing their targets at long range until the Polish army engineer, G. Coriolis, derived the nature of this force in 1835 so that proper corrections could be made.

[4] See "Mass Measurement" (Section 2, this Unit).
[5] See Ealing Film Loop #80-214.
[6] See "Centripetal and 'g' Forces in Circular Motion" (MISN-0-17).
[7] See "Classical Mechanics in Rotating Frames of Reference: Effects on the Surface of the Earth" (MISN-0-18).

3c. Inertial Force is Proportional to Inertial Mass. First note that "$F = ma$" holds for an object in an inertial reference frame, O, where m_i is the object's inertial mass. In a new reference frame O' accelerating with acceleration $\vec{A}$ relative to the first, the observed acceleration will be

$$\vec{a}' = \vec{a} - \vec{A}\,. \tag{3}$$

Yet the applied force, as observed from each frame, is the same (think of the force as being produced by, say, a spring scale). So I can apply Newton's second law in O' and separate off the inertial force due to the acceleration of O': *Help: [S-1]*

$$\vec{F}' = \vec{F} = m_i(\vec{a}' + \vec{A}) = m_i\vec{a}' - \vec{F}_i\,. \tag{4}$$

Thus I have proved that the inertial force, $\vec{F}_i = -m_i\vec{A}$, is proportional to inertial mass.

3d. Einstein's Equivalence Principle. It is important to understand the "real" vs. "fictitious" debate because it was Einstein's genius to notice that inertial forces have one shocking thing proportional to mass! Einstein conjectured that, therefore, the two forces, the "real" gravitational and "fictitious" inertial forces, are, in their essence, one and the same thing. This principle, properly qualified, is Einstein's Equivalence Principle.

3e. Mass Equivalence and Einstein's Equivalence Principle. Because of the equivalence of gravitational and inertial mass, we can make a non-inertial frame equivalent to an inertial one plus a gravity field $\vec{g}$. We can do it by setting the non-inertial-frame field strength $\vec{g}$ equal to the negative of the frame's acceleration $\vec{A}$ with respect to any inertial frame:

$$\vec{g} \equiv -\vec{A}\,.$$

The gravity field works because it, like the inertial force, is proportional to mass alone (given the reference frame we're working in). So using an accelerated reference frame changes our equations of motion in the same way as being in a gravity field![8] Einstein's Equivalence Principle simply states, then, that:

[8] $(-m\vec{A})$ is like the $(m\vec{g})$ in Eq. (2) because $\vec{A}$ and $\vec{g}$ are both constants.

Einstein's Equivalence Principle

The physical effect of a gravity field is indistinguishable (locally) from the effect of using an accelerated reference frame. Conversely, the physical effects of gravity may be (locally) eliminated by using a freely falling reference frame ("locally" is discussed in Sect. 4).

In other words, observers in an accelerating spaceship, far from all masses, feel a force not merely "like" the force of gravity, but indistinguishable from it. And earth-circling astronauts with their rockets turned off seemed free from gravity, so far as local experiments could tell (though they could surely notice out the window that they were circling the Earth!). In this view, the "equal falling" concept of Galileo is explained by the kinematics of acceleration: we view the gravity in terms of an accelerated reference frame. *Help: [S-2]* Thus use of the word "local" and "locally" several places above brings us to Qualifications to the statement of Einstein's Equivalence Principle.

4. Qualifications to the Principle

In a windowless freely falling elevator, is there really *no* way to find out if the Earth's gravity is present? Well, there are at least three ways, but they are all "nonlocal"; they involve comparing measurements in different parts of the elevator, or waiting a while (which is nonlocal in the time domain). The methods are:

1. Set up gravimeters, such as freely falling weights, all over the elevator. Since the elevator cage as a whole responds only to the mean value of the gravity on it, it has at a sort of average acceleration. However, the Earth's gravity is stronger at the bottom and weaker at the top; furthermore, the lines of force converge. Thus weights near the top will appear to fall ever-so-slowly up; near the bottom, down; and near the walls, toward the center. The guy in the cage detects a "tidal" gravity field like that illustrated in the following figure (Fig. 4).

   To understand the sideways arrow on the elevator's right side, think of subtracting the two arrows a and b shown at the right, one of which is the downward acceleration at the center and the other the acceleration at the right, where "down" is slightly different because the Earth's center is not infinitely far away.

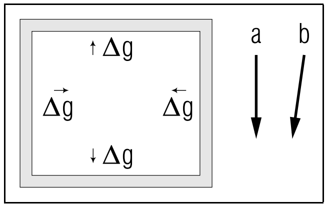

Figure 4. Tidal Gravity in a Falling Elevator. The arrows a and b are accelerations explained in the text.

2. The cage will eventually hit the bottom.
3. A small electric charge suspended in the cage will radiate. (Detection of this involves surrounding the charge with detectors, and is hence nonlocal.) This point is quite sophisticated and the radiation would be too small to detect under any presently envisioned circumstances.

The upshot of all this is that Einstein's Principle of Equivalence is what is called a "local concept"; a sufficiently astute observer can generally find out something about where the masses are if he or she is clever enough. Nevertheless, the Principle is of utmost importance, as it was a guide for formulating the General Theory of Relativity. In that theory, the Newtonian part of the gravity (the two large arrows in Fig. 4) is simply due to choice of reference frame, and only the "tidal" part is uniquely defined independent of frame. This little tidal part tells us the curvature of space-time! But that must be left for another day.

5. Applications of the Principle

We have already seen that the centrifuge is an example where an "artificial" gravity field is created. So it is an application of the Equivalence Principle.

For a fun example, let's find out how fast a 20 m radius Ferris wheel must rotate for the effective g to be zero at the top. Well, the "centrifugal" acceleration (in the wheel frame) is v^2/r. So if $v^2/(20\,\mathrm{m}) = 9.8\,\mathrm{m/s^2}$, the effective g will be zero at the top. This gives $v = 14\,\mathrm{m/s}$, implying rotation at 6.7 rpm. Use a seat belt!

People who want to study the Rayleigh-Taylor instability in fluid mechanics also use the Equivalence Principle. This instability occurs when a heavy fluid floats (rests) on a lighter one. After a while, it will try to settle to the bottom by forming first bulges and then tongues of heavy

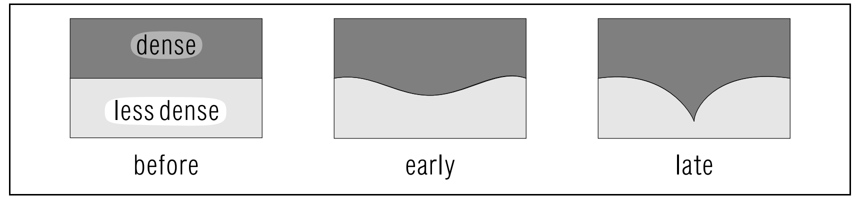

Figure 5. Rayleigh-Taylor Instability.

fluid extending down in to the lighter one below (see Fig.5).

This instability occurs in plasma physics, planetary physics and atmospheric physics. But to study it in the laboratory, how do you ever get the nice smooth layer shown in the first picture? If you try to pour the more-dense on top of the less-dense, the water on top of oil, you get a devil of a mess. The answer is easy: pour the oil on the water and then accelerate the whole container downward! This reverses the effective $\vec{g}$. It is generally done with an explosive charge to get a large upward effective "$\vec{g}$."

6. Active, Passive Gravitational Mass

H. Bondi had pointed out[9] that Newton's law of gravity contains mass both as a source of gravity and as the responsible agent in the particle that responds. These are, respectively, active and passive roles. If particles' active and passive gravitational mass ratios were different for different particles, we could place two such particles near each other and they would exert unequal forces on each other. For example, a particle with an active/passive ratio of zero would attract other particles but would not be attracted to them. This would violate Newton's third law, as would any variation in the active/passive ratio. But all laws are established only to some accuracy, including Newton's third law!

The accuracy of L. Kreuzer's experiment[10] to test the equality of the two kinds of mass is accurate to only four significant digits. And Kreuzer did the best job so far.

It is hard to test the value of "active" gravitational mass, because we have to compare the gravitational force of different laboratory size

[9] H. Bondi, *Rev. Mod. Phys. 29*, 423-428 (1957). The name is pronounced "Bond' ē."

[10] L. B. Kreutzer, *Phys. Rev. 169*, 1007-1012 (1968). The name is pronounced "Kroy'tser."

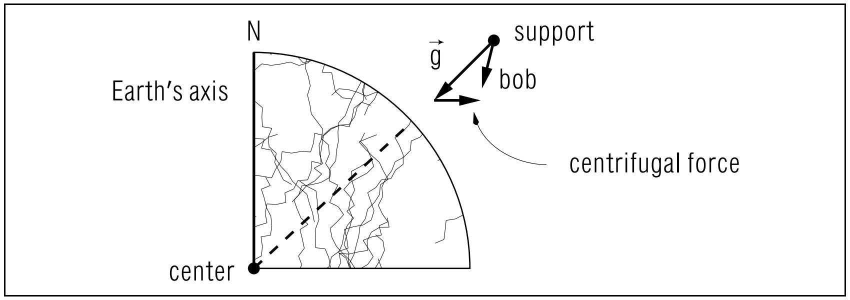

Figure 6. Effect of Earth's Rotation on a Plumb-Bob (Exaggerated)

bodies on each other.[11] Passive gravitational mass is easier to measure accurately, so that is the kind of gravitational mass which is used in precision tests of the equivalence principle.

7. How Exact an Equivalence?

Everything has its limits,[12] and so it was natural for the Hungarian physicist, Baron L. von Eötvös[13] (in collaborations with D. Pekar and E. Fekete) to be inspired to conduct an accurate test[14] of the equality of (passive) gravitational and inertial mass.

Let us try to understand Eötvös's problems in setting up this test. First, note that even if the Earth is perfectly spherical, as we shall assume here, a plumb-bob will not hang vertically (toward the Earth's center) except at the equator or poles. Elsewhere, the downward force of gravity will combine with the centrifugal force away from the Earth's axis[15] to produce a resultant "effective" gravity as shown in Fig. 6. The string will lie along the resultant. Of course, the downward force is proportional to gravitational mass m_{grav} and the little component due to centrifugal force is proportional to inertial mass m_{in}. Hence the angle of the string

[11] See "The Cavendish Experiment" (MISN-0-100).
[12] Including this module, which is nearly over!
[13] Pronounced "et' vuss."
[14] R. V. Eötvös, D. Pekér, and E. Fekete, *Ann. Phys. 68*, 11-16 (1922).
[15] This description is in the rotating frame of the Earth, where the bob is in equilibrium. Note how much more complex is the description in the inertial frame, which avoids "fictitious" forces. In that frame, we say the bob is not in equilibrium, but is accelerated in a circle. The necessary acceleration must be provided by a force component in the string toward the axis of rotation. This gives us the same tilt to the string as just adding real gravity to "fictitious" centrifugal force.

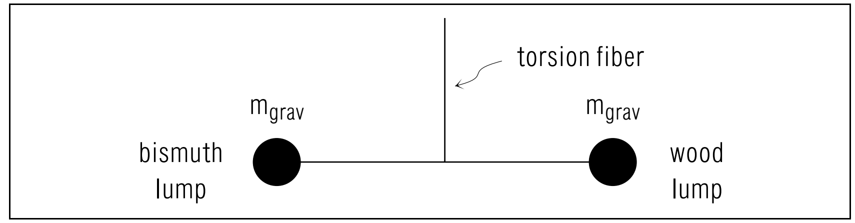

Figure 7. Equal-gravitational-masses torsion pendulum.

depends on the ratio m_{in}/m_{grav}. We can define this ratio to be unity for one substance (say, lead) by choice of Newton's gravity constant,[16] but then we can in principle test the equality for other substances by seeing how plumb-bobs hang when made for them. Clearly, if we try to measure the angles between some long string which have tied to their lower ends lumps of lead, aluminum, soap, and moon rocks, we are going to have a crude (low accuracy) experiment. Eötvös and his collaborators decided to make up some torsion pendulums, each comprised of a fiber holding a thin bar, at the ends of which might be, say, a lump of bismuth and a lump of wood (Fig. 7).[17]

Now, however, we are seemingly no better off than with the plumb bob, because to measure different ratios of m_{in} to m_{grav}, we have to determine how these pendula would hang if the Earth didn't rotate, and compare with the slight possible twist of the fiber when it does. Now we can't stop the Earth. However, each day the Sun's gravity pulls in various directions, and the centrifugal force shown in Fig. 6 has slightly different corrections, at different times of day, for the motion of the Earth around the Sun. At midnight, the Sun's gravity is weakest and the sun-orbit centrifugal force largest, while at noon the opposite holds. *Help: [S-3]* Thus, if the ratio m_{in}/m_{grav} varied slightly from substance to substance, a torsion pendulum using the two substances and oriented with the bar East-West at a temperate latitude would oscillate with a 24 hour period. No such oscillation was found. This enabled Eötvös to set a limit of 5 parts in 10^9 for the variation of the mass ratio in common substances. Later R. H. Dicke[18] of Princeton University and his collaborators P. G. Roll and R. Krotkov improved the apparatus. In 1962 they set a maximal limit of 1 part in 10^{11} for the variation of m_{in}/m_{grav} between gold and alu-

[16] See "Newton's Law of Gravitation" (MISN-0-101).
[17] Eötvös's pendulum also had one weight a bit below the other, for obscure reasons; here we treat a more idealized and superior geometry.
[18] Pronounced "Dick' ē."

minum.[19]

Acknowledgments

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[19]P. G. Roll, R. Krotkov, and R. H. Dicke, *Ann. Phys. (USA) 26*, 442-517 (1964).

PROBLEM SUPPLEMENT

Note: Problems 22-28 also occur in this module's *Model Exam*.

1. How did Newton explain Galileo's law of "equal falling"?

2. Suppose we have two objects of mass m, and we measure the gravity force on them when they are at rest and (a) touching, but not attached, or (b) attached together by a strong but light dab of glue. If the gluing operation does not affect the force of gravity, prove by comparing these situations that gravity force is proportional to mass.

3. Does the above "proof" show that the gravity force depends on mass only? How do you react to the "proof"? Is it too slick? Can you find other situations where such a proof would fail?

4. Suppose we wished to measure the large mass in Fig. 2b in units of the smaller, but had available only one of the small ones, plus standard rockets supplying known thrust (force). How could we make the measurement?

5. An ordinary household spring scale is not used as in Fig. 3a: rather a pointer is attached to the lower end of the spring and is read against a ruled scale. This saves carrying sets of standard weights. Suppose the ruled scale is designed for a location where $g = 9.80\,\mathrm{m/s^2}$. What will be the error in a location where $g = 9.89\,\mathrm{m/s^2}$? Does the balance method of Fig. 3b suffer from this problem?

6. In what kind of reference frame are inertial ("fictitious") forces absent? In what kind are they present?

7. Do you regard centrifugal force as real or fictitious? Is the force that throws you forward when your car stops real or fictitious?

8. Verify the sign in Eqs. (3) and (4).

9. An elevator accelerates up at $4.9\,\mathrm{m/s^2}$. Apply Newton's second law to a 5 kg mass suspended steadily by a spring in the elevator (a) in the earth's frame, and (b) in the elevator frame. Give numerical values for the quantities in Eq. (4) for this case.

10. A repairman examining a merry-go-round that is running has his key chain dangling loosely.

a. If the merry-go-round rotates 0.020 times per second and the man is 3 m from the axis, find the effective gravity he feels.

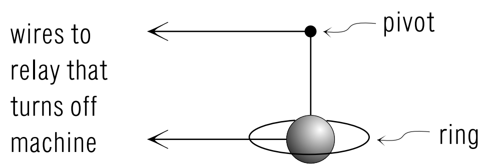
b. At what angle to the vertical does his key case hang?

11. Explain how the picture in Fig. 4 would generalize in three dimensions. How would it differ if the elevator were cylindrical and rotated steadily about a vertical axis as it fell?

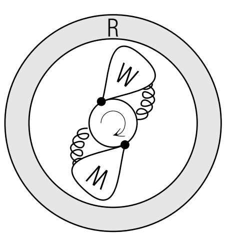
12. Continuing item (11) above: Find a critical speed of rotation for the elevator such that the little arrows toward or away from the axis would vanish in the elevator frame. Hint: the angle θ between arrows "a" and "b" in Fig. 4 is $\theta = r/R$, where r is the distance off axis at which b is measured (a being on axis), and R is the distance (6400 km) to the Earth's center. Because this angle is so small, $|\vec{a} - \vec{b}| \simeq a\theta$.

13. Assuming we can eliminate the small horizontal vectors in Fig. 4 by rotation, how could we still test for the presence of gravity (rather than uniform acceleration)?

14. A pinball machine is designed to turn off and read "TILT" if it is shoved too hard. The sensor for this is a pendulum hung inside a metal ring. It is normally centered, but will hit the ring if the machine is shoved hard, or tilted. Describe the behavior of the TILT detector in the case of a hard shove so as to show the application of the "inertial force" concept.



15. A miniature racer (go-cart) driven by a gasoline engine is advertised to have an "inertial" clutch. The clutch looks like this:



The motor turns the inner shaft as shown. The springs hold the weights "W" toward the shaft. When the shaft turns fast enough, the weights move out and touch the outer race R, driving it by friction, and so moving the cart. Describe the use of inertial forces in this clutch, carefully specifying in what reference frame they appear. Also describe the operation in the rest frame of the cart, emphasizing the action of the spring and its eventual failure to hold in the weights.

16. In Kreuzer's experiment a plastic block floats totally immersed in a liquid carefully chosen to have the same density, in the sense that the float is in neutral equilibrium (tending neither to rise nor sink). Does active or passive gravitational mass determine the densities so compared?[20] The whole apparatus is placed near a sensitive detector of gravity fields and the float moved delicately to and fro. If the detector measure no change in the pull of the float-plus-liquid, does this establish that the float has equal mass with the liquid it displaces? Active or passive mass? Or inertial?)

17. If the acceleration of a falling object can be measured to three significant figures, how accurately does Galileo's experiment verify the Equivalence Principle?

18. Draw a vector diagram showing how to add the Sun's and the Earth's gravity pull acting on Eötvös's pendulum (a) at noon, and (b) at midnight. (Neglect the tilt of the Earth's axis and assume the apparatus is at the equator, so you can work in a plane.)

19. Assuming that you can add the centrifugal force associated with the Earth's motion around the Sun to that associated with the Earth's rotation, and making the same assumptions as in question (18) above, draw vector diagrams showing how to add these two "forces" at noon and at midnight.

20. Describe the situation of a Ferris wheel rotating at the right speed for you to feel "zero gravity" as you pass the top, in terms of the rotating and stationary frames of reference. Which description of the cause of your sensations in simpler, in your opinion? Justify your opinion.

21. Deduce the meaning of the term "tidal" used in Sect. 4 (it does not refer to the earth or ocean tidal forces, but it has a similar meaning relative to a standard uniform background force).

[20] If you are unfamiliar with Archimedes's Principle (MISN-0-48), note that the buoyant force on a submerged, or floating object is equal to the weight of the fluid it displaces.

22. Suppose you ordinarily weigh 150 lb. In an elevator that is starting to move, you notice that your weight is 135 lb. Which way is the elevator starting to go, and what is its acceleration?

23. Assuming Einstein's Principle of Equivalence, prove Galileo's law of "equal falling."

24. When we say that your weight is due to the Earth's mass and yours, in what sense do we mean "mass" for these two objects: inertial, active gravitational, or passive gravitational?

    a. Earth.

    b. You.

25. A 100 gm "weight" (mass = 100 gm) is used to balance an automobile tire. If the weight is 15 cm from the axle and the wheel rotates at $(20/\pi)$ revolutions per second, find:

    a. the centripetal force on the weight in an inertial frame moving with the axle.

    b. the centrifugal force on the weight in the same frame.

    c. the centrifugal force on the weight in a reference frame rotating with the wheel.

    d. the centripetal force in the same frame.

26. Eötvös's original experiment was accurate to about one part in (pick the closest answer):

    a. 100; b. 10^9; c. 10^{12}; d. 10^{25}

    Dicke's improved version was accurate to one part in:

    e. 1000; f. 10^4; g. 10^{11}; h. 10^{40}

27. In a reference frame far from gravitating masses and accelerating with acceleration A, Newton's second law takes the modified form __________ , where a is the acceleration of mass m as measured by the accelerated observer.

28. The Dicke-Eötvös experiment couldn't test the effect of the Earth's centrifugal force because __________ . Instead it looked for variations in the __________ of the Sun and __________ force associated with our motion around it.

Brief Answers:

22. down, $0.98\,\mathrm{m/s^2}$, or $3.2\,\mathrm{ft/s^2}$

23. According to the Principle, we may eliminate the gravity force by using a freely falling reference frame. In this frame, falling objects have no acceleration, as there is no force on them. Therefore, they stay together if initially together, which is Galileo's result. (You can also prove it by replacing the gravity field by an accelerated reference frame in outer space, but the proof is a little longer.)

24. a. active gravitational

    b. passive gravitational

25. a. 24 N

    b. 0

    c. 24 N

    d. 0

26. a. b

    b. b

    c. b

    d. b

    e. g

    f. g

    g. g

    h. g

27. $\vec{F} = m(\vec{a} + \vec{A})$

28. it doesn't vary

    gravity force

    the centrifugal

SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from TX, 3c)*

inertial frame: $\vec{F} = m\vec{a}$.
accelerated frame: total "force" is real one + inertial one:

$$\vec{F}' + \vec{F}_i = m_i \vec{a}'\,.$$

However:

$$\vec{a}' = \vec{a} - \vec{A}$$
$$\vec{F}' = \vec{F}$$

so:

$$\vec{F}_i = -m_i \vec{a}\,.$$

S-2 *(from TX, 3e)*

Replace the locally "constant" gravitational field with a downward-accelerating reference frame ($\vec{a} = \vec{g}$). Then by the equivalence principle there will be no gravitational field present and neither object will move with respect to the other.

S-3 *(from TX, 7)*

The sun is on the side of the earth in sunshine, so the dark side is further from the sun. Further distance means lower gravitational pull and higher centrifugal force ($F_c = m\omega^2 r$ where ω is the same everywhere on the earth).

MODEL EXAM

1. See Output Skills K1-K10 in this module's *ID Sheet*.
2. Suppose you ordinarily weigh 150 lb. In an elevator that is starting to move, you notice that your weight is 135 lb. Which way is the elevator starting to go, and what is its acceleration?
3. Assuming Einstein's Principle of Equivalence, prove Galileo's law of "equal falling."
4. When we say that your weight is due to the Earth's mass and yours, in what sense do we mean "mass" for these two objects: inertial, active gravitational, or passive gravitational?

   a. Earth.

   b. You.
5. A 100 gm "weight" (mass = 100 gm) is used to balance an automobile tire. If the weight is 15 cm from the axle and the wheel rotates at $(20/\pi)$ revolutions per second, find:

   a. the centripetal force on the weight in an inertial frame moving with the axle.

   b. the centrifugal force on the weight in the same frame.

   c. the centrifugal force on the weight in a reference frame rotating with the wheel.

   d. the centripetal force in the same frame.
6. Eötvös's original experiment was accurate to about one part in (pick the closest answer):

   a. 100; b. 10^9; c. 10^{12}; d. 10^{25}

   Dicke's improved version was accurate to one part in:

   e. 1000; f. 10^4; g. 10^{11}; h. 10^{40}
7. In a reference frame far from gravitating masses and accelerating with acceleration A, Newton's second law takes the modified form ______________, where a is the acceleration of mass m as measured by the accelerated observer.

8. The Dicke-Eötvös experiment couldn't test the effect of the Earth's centrifugal force because ____________. Instead it looked for variations in the ____________ of the Sun and ____________ force associated with our motion around it.

Brief Answers:

1. See this module's *text*.
2. See problem 22 in this module's *Problem Supplement*.
3. See problem 23 in this module's *Problem Supplement*.
4. See problem 24 in this module's *Problem Supplement*.
5. See problem 25 in this module's *Problem Supplement*.
6. See problem 26 in this module's *Problem Supplement*.
7. See problem 27 in this module's *Problem Supplement*.
8. See problem 28 in this module's *Problem Supplement*.