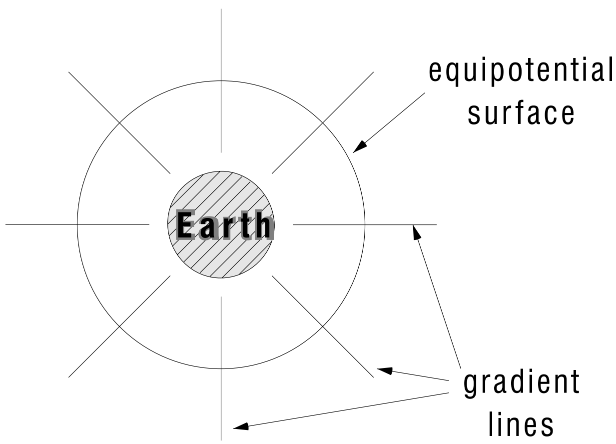
PHYSNET

MISN-0-107

GRAVITATIONAL POTENTIAL ENERGY



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

GRAVITATIONAL POTENTIAL ENERGY

by

Peter Signell and Michael Brandl

Title: **Gravitational Potential Energy**

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Length: 1 hr; 12 pages

Input Skills:

1. Calculate the gravitational potential energy for a mass m at a height h above the surface of the earth (MISN-0-21).
2. State the relationship between radius and angular velocity for circular orbits in an inverse square law force field (MISN-0-102).

Output Skills (Knowledge):

K1. Start from Newton's Law of Gravitation and derive the general expression for gravitational potential energy.

K2. Reduce the general expression for gravitational potential energy to the earth's-surface formula, $E_p = mgh$.

Output Skills (Problem Solving):

S1. Use gravitational potential energy in problems involving conservation of energy.

External Resources (Required):

1. M. Alonso and E. J. Finn, *Physics*, Addison-Wesley (1970). For availability, see this module's *Local Guide*.

Post-Options:

1. "Derivation of Orbits in Inverse-Square-Law Force Fields" (MISN-0-106).
2. "The Gravitational Field" (MISN-0-108).

THIS IS A DEVELOPMENTAL-STAGE PUBLICATION
OF PROJECT PHYSNET

The goal of our project is to assist a network of educators and scientists in transferring physics from one person to another. We support manuscript processing and distribution, along with communication and information systems. We also work with employers to identify basic scientific skills as well as physics topics that are needed in science and technology. A number of our publications are aimed at assisting users in acquiring such skills.

Our publications are designed: (i) to be updated quickly in response to field tests and new scientific developments; (ii) to be used in both classroom and professional settings; (iii) to show the prerequisite dependencies existing among the various chunks of physics knowledge and skill, as a guide both to mental organization and to use of the materials; and (iv) to be adapted quickly to specific user needs ranging from single-skill instruction to complete custom textbooks.

New authors, reviewers and field testers are welcome.

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A. Introduction

The Universal Law of Gravitation describes a force between masses. Any work done in changing a configuration of masses thus results in a change in potential energy. The formula for this energy is derived and used, particularly in problems involving conservation of energy.

B. Study Material

In AF[1] read sections 15.4 and 15.5, noting that the word "grad" in the fourth line, second column, page 310, should be in bold face type.

Work through the "Proof of relation (15.3)" on page 310, and Examples 15.3 and 15.4.

Work problems 15.17, 15.19 and 15.23.

C. Brief Answers to Problems

15.17: 2.62×10^{33} J, -5.33×10^{33} J, -2.62×10^{33} J

15.19: 4.32×10^{3} m/s, 1.03×10^{4} m/s, 5.06×10^{3} m/s, 6.03×10^{4} m/s.

15.23: 1.02×10^{4} m/s, from $v = \sqrt{5\gamma M_E/3R_E}$.

D. Exercises

1. Figure out why one uses conservation of energy in the assigned problems rather than, say, getting the velocities from $v = v_0 + at$.

2. Derive Newton's Law of Gravitation from the expression for gravitational potential energy by applying the gradient operator. For you skiers, the gradient of a function points up the function's fall line.

[1] M. Alonso and E. J. Finn, *Physics*, Addison-Wesley (1970). For availability, see this module's *Local Guide*.

Note that, in two dimensions, the gradient operator is:

$$\vec{\nabla} \equiv \mathbf{grad} \equiv \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta}\,.$$

Now:

$$\partial E_p(r)/\partial\theta = 0,$$

so:

$$\vec{\nabla} E_p(r) \equiv \mathbf{grad} E_p(r) = \frac{dE_P(r)}{dr}\,\hat{r}\,.$$

Acknowledgments

Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

LOCAL GUIDE

The readings for this unit are on reserve for you in the Physics-Astronomy Library, Room 230 in the Physics-Astronomy Building. Ask for them as "The readings for CBI Unit 107." Do **not** ask for them by book title.

PROBLEM SUPPLEMENT

PHYSICAL AND ORBITAL DATA FOR THE SOLAR SYSTEM*				
Body	Mean Radius[c] (meters)	Mass (kg)	Mean Orbital Radius (meters)	Orbital Period (seconds)
Sun	6.96×10^8	1.99×10^{30}	- - - - - -	- - - - - -
Mercury[a]	2.43×10^6	3.30×10^{23}	5.79×10^{10}	7.60×10^6
Venus[a]	6.06×10^6	4.87×10^{24}	1.08×10^{11}	1.94×10^7
Earth	6.37×10^6	5.98×10^{24}	1.50×10^{11}	3.16×10^7
Mars[a]	3.37×10^6	6.40×10^{23}	2.28×10^{11}	5.94×10^7
Jupiter	6.99×10^7	1.90×10^{27}	7.78×10^{11}	3.74×10^8
Saturn	5.84×10^7	5.69×10^{26}	1.43×10^{12}	9.30×10^8
Uranus	2.30×10^7	8.73×10^{25}	2.87×10^{12}	2.65×10^9
Neptune	2.22×10^7	1.03×10^{26}	4.50×10^{12}	5.20×10^9
Pluto	$< 3 \times 10^6$	$< 6 \times 10^{23}$	5.90×10^{12}	7.82×10^9
Moon[b]	1.74×10^6	7.35×10^{22}	3.84×10^8	2.36×10^6

*Data adapted from the Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac, Her Majesty's Stationery Office, London (1961). Notes: [a]Radius obtained from radar observations, mass from space probe perturbations. [b]Orbital data are with respect to the earth. [c]Radius of a sphere of equal volume.

Work the AF problems listed in this module's *text*.

1. Start from Newton's Law of Gravitation and derive the general expression for gravitational potential energy.

2. A projectile is fired vertically from the earth toward the moon.

   a. At what point on its path toward the moon will its acceleration be zero?

   b. What would the minimum muzzle velocity (initial velocity of the projectile) need to be to allow the projectile to reach that point, and then fall toward the moon under the influence of the moon's gravity. Assume air resistance is negligible.

   c. In this case, what would the projectile's velocity be when it hit the moon?

3. Reduce the general expression for gravitational potential energy to the earth's surface formula $E_p = mgh$.

Brief Answers:

1. $F = -\dfrac{Gmm'}{r^2}\,\hat{r}$ (Newton's Law of Gravitation).

   Designating ($r = \infty$) as the potential energy reference point, $E_p(\infty) = 0$, and denoting the radial component of $\vec{F}$ as F_r:

$$E_p(r) = -\int_\infty^r \vec{F}\cdot d\vec{r} = -\int_\infty^r F_r\,dr = \int_\infty^r \frac{Gmm'}{r^2}\,dr$$

$$= Gmm'\int_\infty^r r^{-2}\,dr = -\frac{Gmm'}{r}\,.$$

2. a. $x = \dfrac{d_{ME}}{1+\sqrt{m_M/m_E}}$.

   b. $V_0 = \sqrt{2G\left(\dfrac{m_E}{R_E} + \dfrac{m_M}{d_{ME}-R_E} - \dfrac{m_E}{x} - \dfrac{m_M}{d_{ME}-x}\right)}$.

   c. $v_f = \sqrt{2G\left(\dfrac{m_E}{d_{ME}-R_M} + \dfrac{m_M}{R_M} - \dfrac{m_E}{x} - \dfrac{m_M}{d_{ME}-x}\right)}$.

   NOTE: Some of the above terms are negligibly small.

3. $\Delta E_p = E_p(R_E+h) - E_p(R_E) = -\dfrac{Gmm_E}{R_E+h} + \dfrac{Gmm_E}{R_E} = \dfrac{Gmm_E h}{R_E(R_E+h)}$

   At earth's surface:

$$F = mg = \frac{Gmm_E}{R_E^2},\ \text{so: } Gm_E = gR_E^2$$

   Putting this into the above,

$$\Delta E_p = \frac{mghR_E}{R_E+h} = mgh\frac{1}{1+h/R_E}$$

   Since the quantity in the parentheses will be negligibly changed by exclusion of (h/R_E), providing ($h \ll R_E$), we get:

$$\Delta E_p \approx mgh, \qquad h \ll R_E.$$

   If we change our potential energy reference:

$$E_p(h) \approx mgh, \qquad h \ll R_E.$$

MODEL EXAM

PHYSICAL AND ORBITAL DATA FOR THE SOLAR SYSTEM*				
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   c. In this case, what would the projectile's velocity be when it hit the moon?

3. Reduce the general expression for gravitational potential energy to the earth's surface formula $E_p = mgh$.

Brief Answers:

1. See this module's *Problem Supplement*, problem 1.

2. See this module's *Problem Supplement*, problem 2.

3. See this module's *Problem Supplement*, problem 3.