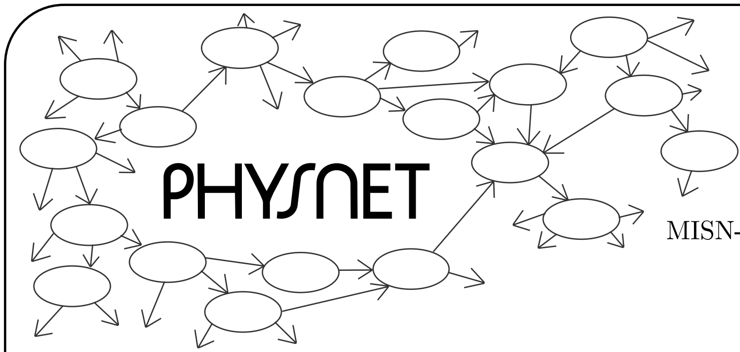
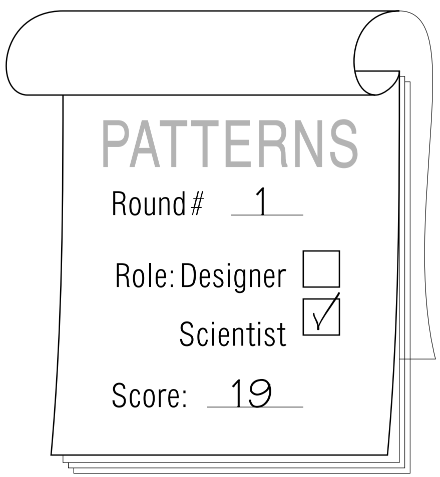


MISN-0-70

INDUCTIVE REASONING: THE GAME OF PATTERNS



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

INDUCTIVE REASONING: THE GAME OF PATTERNS

by

Richard McCoy

Title: **Inductive Reasoning: The Game of Patterns**

Author: Richard McCoy, Michigan State University

Version: 2/22/99 Evaluation: Stage 0

Length: 1 hr; 12 pages

Input Skills:

1. Some familiarity with Newton's Law of Universal Gravitation (MISN-0-410) or (MISN-0-101).

Output Skills (Knowledge):

K1. Describe the difference between inductive and deductive reasoning.

K2. State why induction plays an important role in the formulation of scientific theories.

Output Skills (Problem Solving):

S1. Draw and recognize patterns which exhibit some type of orderliness such as symmetry.

Output Skills (Project):

P1. Play at least two rounds of the Game of Patterns successfully.

External Resources (Required):

1. Magazine Article: M. Gardner, "Mathematical Games," *Scientific American*, New York (Nov. 1969), pp. 140-144, or M. Gardner, *Mathematical Circus*, Alfred A. Knopf, New York (1979), pp. 45-55, "Patterns of Induction." For access to these resources, see the *Local Guide* at the end of this module.

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The goal of our project is to assist a network of educators and scientists in transferring physics from one person to another. We support manuscript processing and distribution, along with communication and information systems. We also work with employers to identify basic scientific skills as well as physics topics that are needed in science and technology. A number of our publications are aimed at assisting users in acquiring such skills.

Our publications are designed: (i) to be updated quickly in response to field tests and new scientific developments; (ii) to be used in both classroom and professional settings; (iii) to show the prerequisite dependencies existing among the various chunks of physics knowledge and skill, as a guide both to mental organization and to use of the materials; and (iv) to be adapted quickly to specific user needs ranging from single-skill instruction to complete custom textbooks.

New authors, reviewers and field testers are welcome.

INDUCTIVE REASONING: THE GAME OF PATTERNS

by

Richard McCoy

1. Two Methods of Reasoning

1a. Studying Our Reasoning Ability. One of our most valuable tools is the ability to reason. On the one hand, it allows us to generate theories which can generalize and predict; on the other, it enables us to use such theories to solve specific problems. In fact, all knowledge that does not come directly from the senses is the end product of some reasoning process. Thus it is important that we understand various reasoning processes, particularly with regard to the nature and value of the conclusions they yield.

1b. Deductive and Inductive Reasoning. There are two general methods of reasoning used in scientific work: deductive reasoning and inductive reasoning. In deductive reasoning, one starts with a set of premises and formulates a conclusion whose truth is guaranteed to the extent of the truth of the premises. Therefore, in a deductively valid argument, it is impossible for both the premises to be entirely true and the conclusion in any way false.[1] In contrast, inductive reasoning uses true facts (reproducible observations) to infer statements about what is probably true of the world in general. Since our factual knowledge of the world can never be complete, we can never be sure that we have absolute inductive truth.

1c. Each Has Its Uses. In practice, each form of reasoning, inductive and deductive, has its uses. Deductive reasoning is most often used to solve particular problems; that is, to go from the general to the specific. Thus, if one makes the assumption that Newton's Law of Gravitation is true (i.e., that $F = GM_1M_2/R^2$), it can be deductively predicted that the inter-mass force will become four times as great if the radius is halved. Here, if the assumption (premise) is true, the conclusion must inevitably be true. Inductive reasoning, however, is most often used to make generalizations, or predictive theories, from a set of specific facts which can often be scientific data or evidence. Here, the premises, or evidence, are observations such as "I have seen ninety-nine crows and they are all black," and the conclusion is a probabilistic statement such as "The next crow I see will almost certainly be black" or "Every crow is black, in all likelihood." It is possible that a future observation will disprove the conclusion, but the more evidence that is accumulated in support of the conclusion the more probable it is that the conclusion is true.[2] Scientific research has made good use of both types of reasoning process. The normal investigative procedure is to examine current knowledge of a subject and form a hypothesis that would account for it. This is an inductive procedure, yielding a conclusion (hypothesis) that could easily be false. The scientist next uses the hypothesis as a premise in a deductive argument to predict what will happen under previously unexplored conditions. Having obtained the predictions, the scientist tests them by creating these conditions and seeing if the results confirm the predictions. If the predictions are correct, the hypothesis is more likely to be true, while if the predictions are incorrect the hypothesis is discarded or revised. It is apparent that theoretical (deductive) advances in the sciences have rested in great part upon the inductive process of generalizing from experience.

[1] See Appendix A for a discussion of the terms "argument" and "premise."

2. The Game of Patterns

Martin Gardner has published commentary, rules, and examples of Sydney Sackson's game of Patterns that demonstrates the inductive reasoning process.[3] To play this game you must recruit two other players. All three members of your group should read the Gardner material carefully. Then, after all of you are satisfied with your understanding of the rules (don't hesitate to discuss any questions you may have with your partners - this is a cooperative project), play the game a few times, making sure you employ cooperative learning methods as you go along in order to help yourself and your partners.[4]

[2] The example of predicting that the 100th crow will be black, given that the first ninety-nine were black, is known as Leibniz's Law, and is interesting in that, under minimal assumptions, it can be concluded that the probability of the 100th crow being black is at least 0.99.

[3] M. Gardner, "Mathematical Games," *Scientific American*, November 1979, pp. 140-144 and/or M. Gardner, *Mathematical Circus*, Alfred A. Knopf, New York (1979), pp. 45-55, "Patterns of Induction."

[4] Forms for playing the game are attached.

3. Cooperative Learning as a Tool

3a. Strategy Discussions. After each round of the game, discuss your strategy with your partners, striving to improve your collective grasp of inductive learning skills. A good indicator of your skill is, of course, your score. However, don't play "cut throat"—seeking only to beat your partners' scores. Try instead to help the others improve their scores by suggesting how they can improve their play and accepting suggestions from them. This means, of course, no put-downs. Make all your criticism constructive.

3b. The Role of Designer. The role of Designer (or Nature or the Universe or the Deity), for the purpose of learning about inductive reasoning, should be different than what it would be if you were simply playing for recreation. As Designer, make patterns that are difficult enough to make your partners think, yet not so tough that the game becomes a chore. Do not try for high scores as Designer; but rather, use this role to aid your partners in improving their understanding.

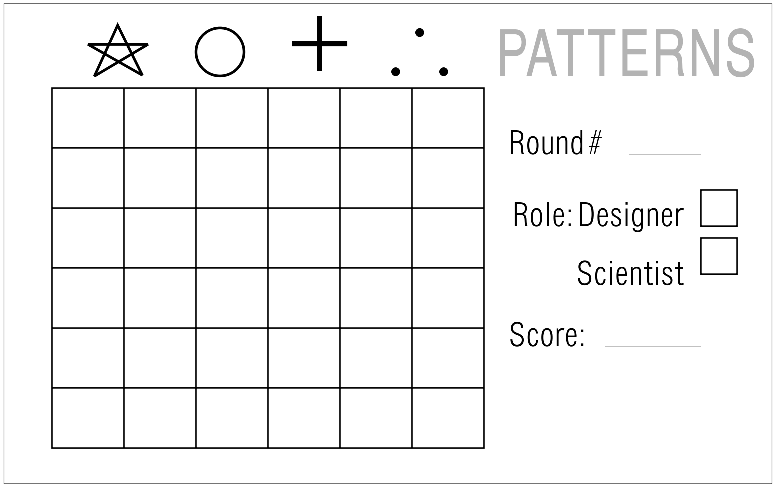
Acknowledgments

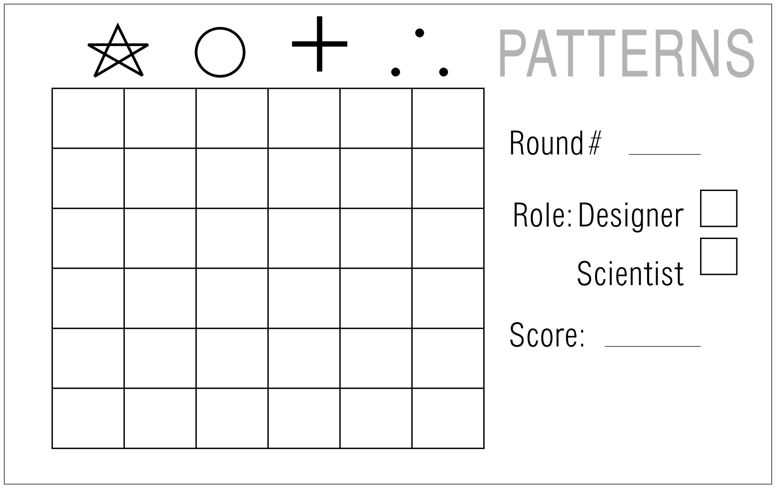
I wish to thank Ken Franklin for writing a prior module and for many discussions. Production of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

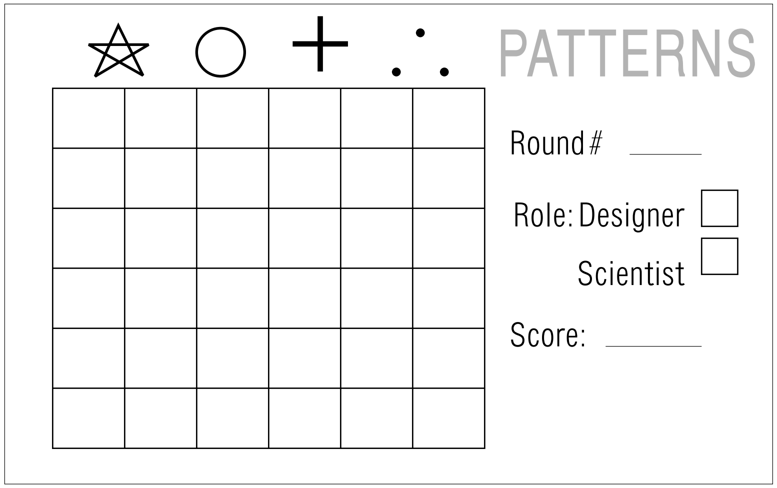
A. Terminology of Reasoning

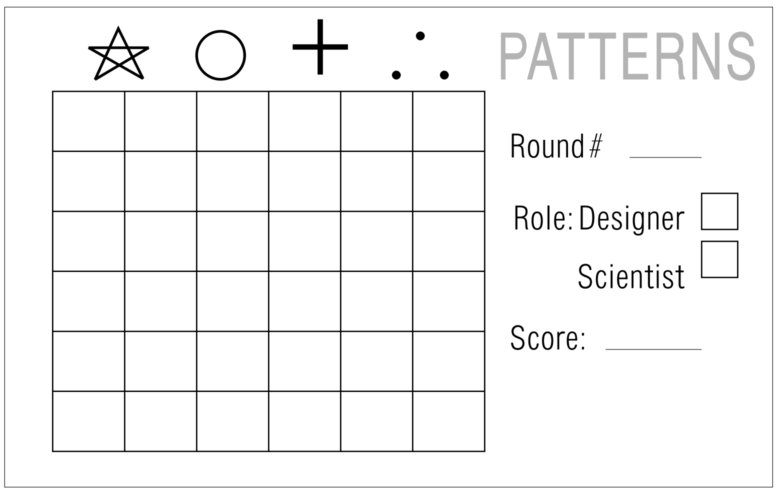
An argument, when used in the context of induction and deduction, consists of (1) a set of premises, (2) a conclusion, and (3) the reasoning that links premises to conclusion. In deductive reasoning the premises can be axioms established by agreement, factual statements, or empirical observations and data. Induction, on the other hand, almost exclusively employs empirical observations and data as premises. For this reason, the premises in an inductive argument are usually referred to as "evidence."

B. Game Sheets









LOCAL GUIDE

The readings cited on the "ID Sheet" and in the footnotes are on reserve for you in the Physics-Astronomy Library, Room 230 in the Physics-Astronomy Building. Ask for them at the counter as "the readings for CBI Unit 70."

You must bring the originals of your completed game sheets, not copies, to the Exam Room when you come to take your exam, and you must staple the game sheets to your Exam Answer Sheet(s).

MODEL EXAM

Note to Examinee: staple the completed game sheets to your Exam Answer Sheet(s).

Note to Examiner: Make reasonably sure the game was actually played.

1. Answer these questions "yes" or "no":
   a. Did your learning partners engage in constructive criticism after each round of Patterns, giving you suggestions on how you could better apply induction techniques to its play?
   b. Did either of your partners concern themselves more with winning the game than improving your group's collective understanding of inductive reasoning?
   c. Have you and your partners played at least two games of Patterns?
2. a. Describe the difference between inductive and deductive reasoning.
   b. Why does induction play an important role in the formulation of scientific theories?