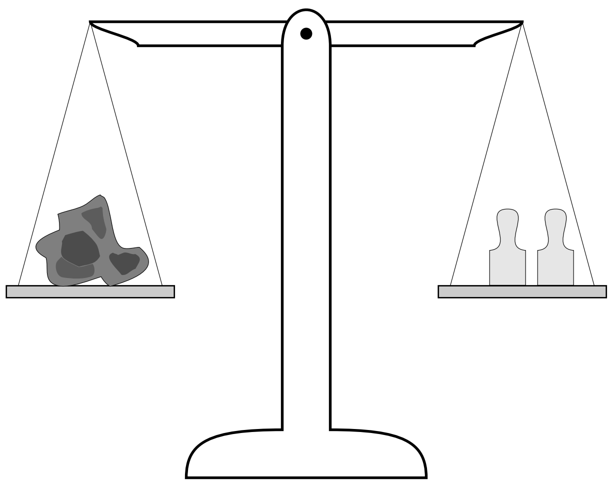
PHYSNET

MISN-0-64

MASS AND WEIGHT



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

MASS AND WEIGHT

by

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Title: **Mass and Weight**

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Input Skills:

1. A knowledge of the three laws of motion (MISN-0-409).

Output Skills (Knowledge):

K1. Define mass and describe a method for determining the mass of an unknown object in terms of a standard mass.

K2. Define weight and state at least two differences between mass and weight.

Output Skills (Rule Application):

R1. Determine the mass of an unknown object from experimental data describing its interaction with an object of known mass.

R2. Given g, find an object's mass from its weight and vice versa.

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1. Introduction

You probably already have some idea of the meaning of the word "mass"; you certainly use the word "massive." You may even have measured the mass of an object using a balance. You also have an idea what "weight" is; however, you may well be somewhat unclear about the difference between weight and mass. In this unit, you will learn the very precise definitions these terms have in physics and the very clear distinctions between the two terms, "mass" and "weight."

2. Mass

2a. The Definition of Mass. You will recall that the mass of an object is defined to be the constant of proportionality in Newton's second law when applied to the object:[1]

$$\text{(force on object)} = \text{(mass of object)} \times \text{(acceleration of object)}$$

or:

$$\vec{F} = m\vec{a}\,.$$

From this definition you can see that a larger mass requires a larger force to achieve the same acceleration. Thus mass is related to a "resistance" to acceleration, an inertia. Sometimes m is called the object's "inertial" mass.

2b. Some Properties of Mass. It turns out that mass is an intrinsic property of an object. It does not depend on where the object is or on any other of a long list of environmental factors—lighting, temperature, etc. For example, the mass of a golf ball turns out to be the same in the dark as in the light, the same on a hot day as on a cold day, and the same on the moon as on the earth.

2c. Mass Comparisons. Newton's second and third laws of motion can be combined to give a method for comparing the masses of different

[1]See "Particle Dynamics - Newton's Laws" (MISN-0-409).

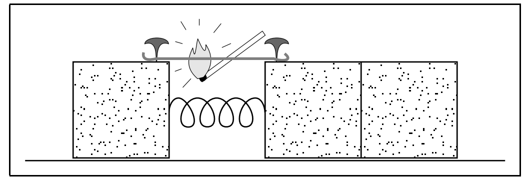

Figure 1. An experimental arrangement for comparing masses.

objects. Figure 1 shows an experimental method which can be used for this purpose. In this particular case, a very light spring is squashed between three identical blocks as shown.

The blocks are placed on a nearly frictionless surface and are held together by a string. When the string is burned, you find that the magnitude of the acceleration of the double block is half that of the single block. This result is independent of the strength of the spring, the degree of squashing, etc. The interpretation of this experiment is relatively simple:

- The third law of motion states that the force on the single block is equal and opposite to the force on the double block.
- The second law states that the mass times the acceleration of a set of blocks equals the force on it.
- Combining these two statements gives the result that the mass of the double block is twice the mass of the single block.

You can do this experiment for any possible combination of blocks. In every case, you will find that the mass of the object is proportional to your intuitive concept of the "quantity of matter" that is being accelerated.

By picking one object to act as a standard of mass, m_{st}, the mass of any other object m_{ob} can be determined in terms of that standard: all one need do is allow the unknown mass to interact via a spring with the standard mass. In terms of the measured acceleration of the two masses;

$$m_{ob} = m_{st} \cdot \frac{a_{st}}{a_{ob}}.$$

The verification and an application of this result is left as these two exercises:[2]

[2]If you need help with an exercise, turn to the appropriate sequence in this module's *Special Assistance Supplement*.

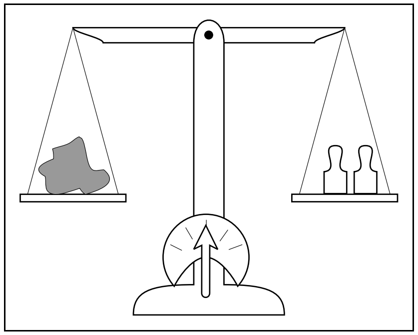

Figure 2. An analytical balance.

▷ Prove the relation stated above. [S-1]

▷ An unknown mass and a 5 kilogram mass interact in the manner described above. The unknown mass has an acceleration of $5\,\mathrm{m/s^2}$ while the known mass has an acceleration of $7\,\mathrm{m/s^2}$. What is the mass of the unknown object? [S-2]

2d. The Above Method is Not Used. Although the procedure for measuring masses that is given above is often quoted as the way by which the mass scale is defined, this method is hardly ever used in practice. Some of the reasons are:

- it is difficult since frictionless surfaces are hard to find;
- it is not very precise since accelerations are very hard to measure with any degree of precision;
- it is not very practical since many objects would break or spill in the process;
- there exists a much more convenient, precise technique for measuring masses.

2e. Mass Measurements. A convenient way to measure masses is to use an analytic balance as in Fig. 2. In use, carefully calibrated masses are added to the known side until the apparatus balances. Then the mass of the unknown can be found by summing the known masses. This device actually compares the gravitational masses of the objects, not their inertial masses. However, it has been found that gravitational mass and inertial mass are the same.[3]

[3] See "Relativistic Gravitation: The Equivalence Principle" (MISN-0-110), where the current accuracy in measurements of the equivalence of the two kinds of mass is quoted.

3. Weight

3a. The Acceleration of Free Fall. Experimentally it is found that all freely falling objects near the surface of the earth have the same acceleration, roughly $9.8\,\mathrm{m/s^2}$ (or $32.2\,\mathrm{ft/s^2}$). This is true no matter what material the object is made of or how large the object is - at least to the extent that the effects of air resistance can be ignored. This acceleration of freely falling objects is usually designated as g or $\vec{g}$ and is called the "acceleration of gravity."[4] The direction of $\vec{g}$ is toward the center of the earth. The cause of this acceleration is discussed elsewhere.[5]

3b. Weight Defined. The force that causes a freely falling object to accelerate is commonly called the "weight" of the object. Newton's second law says that if an object has acceleration $\vec{g}$, then the force on it is given by:

$$\vec{F} = m\,\vec{g}\,.$$

Thus the weight of an object is defined by:

$$\vec{F}_w = m\,\vec{g}\,. \tag{1}$$

The expression "the weight of an object" is just a short way of saying "the force of gravity on the object."

3c. Some Properties of Weight. The weight of an object depends on both the mass of the object and the acceleration of gravity [see Eq. (1)]. The acceleration of gravity is different at different places on the surface of the earth. It also varies with altitude and is much different on the moon. In summary, the weight of an object varies from place to place while its mass does not; you have a smaller weight on the moon than on the earth, but your mass is the same in both places. The following exercise emphasizes this fact:

▷ A person weighs 212 pounds on earth, where $g = 32.2\,\mathrm{ft/s^2}$. What would this person weigh at a place where $g = 10.0\,\mathrm{ft/s^2}$? [S-3]

3d. The Meaning of "Weight" as a Word. The physicist's use of the word "weight" is quite precise: weight is a local gravitational force. In everyday usage, for contrast, weight may refer to either force or mass. For example, people sometimes quote an object's "weight" in both pounds and

[4] The use of the term "acceleration of gravity" is somewhat weird since it is the object and not gravity that is accelerating.

[5] See "The Gravitational Force" (MISN-0-410).

kilograms, not realizing that pounds are units of force while kilograms are units of mass. Even some physicists sometimes use the terms that way.

There are two things you can do to help yourself in this confusing situation. First, avoid use of the word "weight" in technical discussions: use mass, or force of gravity, or some other term with a clear meaning. Second, whenever you come across the word "weight" in a discussion, ask yourself whether the person is referring to force or mass: the units will usually tell you.

4. Units

4a. The SI Units. In the SI (metric) system of units, mass is measured in kilograms and weight is measured in newtons. A liter of water (roughly a quart) has a mass of one kilogram and, at the surface of the earth, a weight of 9.8 newtons.

4b. The cgs Units. In the cgs (centimeter-gram-second) system of units, mass is measured in grams and weight in dynes. A nickel has a mass of about 5 grams and, at the surface of the earth, a weight of about 5000 dynes. An object traditionally quoted as having a weight of 1 dyne (at the surface of the earth) was a fly's wing (or was it a gnat's eyebrow?). Note that in the cgs system of units the abbreviation for the gram is 'gm' whereas in SI units the abbreviation for the same quantity is 'g'.

4c. The English Units. In the English system of units, mass is measured in slugs and weight in pounds. One slug has a weight of 32.2 pounds at the surface of the earth.

The common legal unit of mass in the U.S. is the "pound avoirdupois," abbreviated "lb avdp," defined as:

$$\text{lb avdp} = \frac{\text{slug}}{32.1739}\,.$$

Thus an object with a mass of one lb avdp has a weight of one lb at the surface of the earth.[6]

4d. Conversion Factors and Summary. Here are a few useful mass conversion factors:

$$\begin{aligned} 1000\text{ grams} &= 1\,\text{kg} \\ 1\text{ lb avdp} &= 0.4535924277\,\text{kg} \\ 1\text{ slug} &= 9.80665\text{ kg} \end{aligned}$$

[6]The "pound avoirdupois" is pronounced "pound av-rd-uh-poiz."

Here are two tables that summarize the various sets of units:

	mks system		English system	
Quantity	Unit	abb	Unit	abb
mass	kilogram	kg	slug pound avoirdupois	(none) lb avdp
weight	newton	N	pound	lb

	cgs system	
Quantity	Unit	abb
mass	gram	gm
weight	dyne	dyn

Acknowledgments

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PROBLEM SUPPLEMENT

For g at the earth's surface use $9.8\,\mathrm{m/s^2}$ or $32.2\,\mathrm{ft/s^2}$.

To get maximum benefit from the practice problems, you should make a determined effort to solve them yourself. If you cannot, after considerable struggle, then you may be able to find help in a sequence in this module's *Special Assistance Supplement*. For example, a reference marked [S-4] means that you can find special help in sequence [S-4] in the *Special Assistance Supplement*.

Problems 7-8 also occur in this module's *Model Exam*.

1. A student is working in a physics laboratory with two essentially frictionless cars arranged as shown above. One car is a standard car and has a mass of 1.5 kg. The compressed spring is not connected to either car. When the string is burned, the acceleration of the 1.5 kg car is $6.0\times10^1\,\mathrm{cm/s^2}$, while the acceleration of the other car is $4.0\times10^1\,\mathrm{cm/s^2}$. What is the mass of the second car?

2. The same problem as above, only this time the standard car has a final velocity of 3.0 ft/s while the other car has a final velocity of 9.0 ft/s. What is the mass of the second car this time?

3. Yet another student does the same experiment. She observes that the standard car goes 8.0 inches in the same time interval that the other car goes 2.0 feet. What is the mass of the second car?

4. Two students are eagerly working in the laboratory. They allow two air pucks connected by a rubber band to snap together, starting from rest. One, a 2.0 kg puck, attains a final velocity of 6.0 m/s, the other, 2.0 m/s. What is the mass of the second puck?

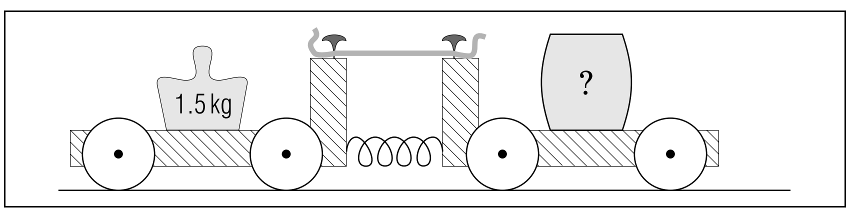

Figure 3. The experimental arrangement for Problems 1-3.

5. The Krystal Klear Ice company has sent a salvage crew to the asteroid belt to retrieve a solid ice asteroid which they hope to be able to sell. Their chief mechanic is installing a two-cable anchor on the asteroid. On earth, the anchor has a weight of 17,760 pounds.

   What is its mass on the asteroid, if the acceleration of gravity is $1.00\times10^{-1}\,\mathrm{ft/s^2}$ there? What is its weight on the asteroid? If the mechanic applies a force of 125 pounds to it, how fast does it accelerate?

6. A recent astronaut has a mass of 75 kg on the earth. Determine the astronaut's weight:

   a. at 140 miles above the surface of the earth where $g = 9.17\,\mathrm{m/s^2}$;

   b. on the surface of the moon where $g = 1.62\,\mathrm{m/s^2}$.

   c. Determine the astronaut's mass in each case.

7. A socket wrench set has a weight of 16 pounds on the earth. What is its mass on the moon, where the acceleration of gravity is only 1/7 that of the earth?

8. Two high-quality toy cars having almost no friction are pushed apart by a spring. Both cars start from rest and interact only with each other via the spring and with the board on which they are rolling. One car has a mass of 1 kg and travels 15 m in 10 seconds. The other car goes 10 m in the same 10 second interval. What is the mass of the second car?

Brief Answers:

1. 2.3 kg

2. 0.50 kg [S-4]

3. 0.50 kg [S-5]

4. 6.0 kg

5. 552 slugs, 55.2 lbs, $0.226\,\mathrm{ft/s^2}$ [S-6]

6. (a) 688 N; (b) 122 N; (c) 75 kg, 75 kg

7. 1/2 slug.

8. 1.5 kg.

SPECIAL ASSISTANCE SUPPLEMENT

Purpose.

This part of the unit will provide you with additional assistance in mastering the concepts presented in the unit and in applying them to the problems. You need not study this material if you feel you have mastered the text material and if you have successfully solved the exercises and problems without assistance.

A Drill on Weight and Mass.

If you can clearly distinguish between weight and mass, you should be able to answer the following questions. The answers are at the end of the list, and a full discussion of these items is found in the text.

1. Mass is a (vector/scalar) ________________.
   Weight is a (vector/scalar) ________________.
2. The direction of mass is ________________.
   The direction of weight is ________________.
3. The mass of an object depends on the nature of the object. (T/F)
   The mass of an object depends on where the object is located. (T/F)
4. The weight of an object depends on the nature of the object. (T/F) The weight of an object depends on where the object is located. (T/F)
5. The units of mass are ________________. (There are three.)
6. The units of weight are ________________. (There are three.)
7. The weight (on earth) of 1 kg mass is ________________.
8. The weight of 1 slug mass (on earth) is ________________.

Brief Answers.

1. Scalar. Vector.
2. Mass has no direction. Weight is downward.
3. True. False.
4. True. True.
5. Grams, kilograms, or slugs.
6. Dynes, newtons, or pounds.
7. 9.8 newtons.
8. 32.3 pounds.

Special Assistance Sequences.

S-1 *(from TX-2c)*

We use the subscripts *ob* to refer to the object and *st* to refer to the standard. Newton's third Law says

$$\vec{F}_{ob} = -\vec{F}_{st},$$

but we are not interested in directions, only magnitudes so this is:

$$F_{ob} = F_{st}$$

Newton's second law says (in one dimension)

$$F_{ob} = m_{ob}\, a_{ob}$$

and

$$F_{st} = m_{st}\, a_{st}.$$

Combining

$$m_{ob}\, a_{ob} = F_{ob} = F_{st} = m_{st}\, a_{st}.$$

Solving:

$$m_{ob} = m_{st}\frac{a_{st}}{a_{ob}}\,.$$

S-2 *(from TX-2c)*

We plug into the formula:

$$\begin{aligned} m_{ob} &= m_{st}\frac{a_{st}}{a_{ob}} \\ &= (5\,\text{kg})\,(7\,\text{m/s}^2)/(5\,\text{m/s}^2) \\ &= 7\,\text{kg}. \end{aligned}$$

S-3 *(from TX-3b)*

The crucial concept here is that the mass is the same in both cases. Using W for weight and a subscript e to indicate the earth:

$$W_e = m\,g_e$$

or

$$m = W_e/g_e\,.$$

Using a subscript t to mean "there":

$$W_t = m\,g_t\,.$$

Combining,

$$\begin{aligned} W_t &= W_e g_t/g_e, \\ &= (212\,\text{lb})\,(10.0\,\text{ft/s}^2)\,/\,(32.2\,\text{ft/s}^2) \\ &= 65.8\,\text{lb}. \end{aligned}$$

S-4 *(from PS-Problem 2)*

This problem is really the same as in [S-1] and [S-2] with a few new twists. The only new point is that you must recognize that while the third law says the ratio of the acceleration of the two cars is always the same, the connection between velocity and acceleration means the velocities always have the same ratio (as do the distances traveled during any fixed time interval). In concrete terms, if two objects start from rest and the first always has 3 times the acceleration of the other, then the first one will always travel 3 times as far in any specific time interval. The upshot of this is that you can use the velocity ratios to calculate the masses - using magnitudes only:

$$m_{st}\,v_{st} = m_{ob}\,v_{ob}$$

$$\begin{aligned} m_{ob} &= \frac{v_{st}}{v_{ob}}m_{st} \\ &= 0.5\,\text{kg} \end{aligned}$$

Note that since you are taking a ratio of the velocities, the units of ft/s drop out.

S-5 *(from PS-Problem 3)*

Another similar problem. All the comments above apply, except that this time you use a ratio of distances and you must worry about the mixed units since they do not drop out.

$$m_{st}\,d_{st} = m_{ob}\,d_{ob}\,.$$

$$\begin{aligned} m_{ob} &= m_{st}\frac{d_{st}}{d_{ob}}\,, \\ &= (1.5\,\text{kg})\left(\frac{8\,\text{inches}}{2\,\text{ft}}\right)\left(\frac{1\,\text{ft}}{12\,\text{inches}}\right)\,, \\ &= 0.5\,\text{kg}\,. \end{aligned}$$

S-6 *(from PS-Problem 5)*

This is a very standard type of problem used to see if you can distinguish between mass and weight. From the weight of the object on the earth, we find its mass:

$$\begin{aligned} m &= \frac{w}{g} \\ &= \frac{17760\,\text{lb}}{32.2\,\text{ft/s}^2} \\ &= 551.6\,\text{slugs}\,. \end{aligned}$$

The mass on the asteroid is the same. However, its weight there is:

$$\begin{aligned} w &= m\,g \\ &= (551.6\,\text{slugs})\,(0.100\,\text{ft/s}^2) \\ &= 55.2\,\text{lb}. \end{aligned}$$

The acceleration is obtained from the second law of motion (using only magnitudes):

$$\begin{aligned} a &= \frac{F}{m} \\ &= \frac{125\,\text{lb}}{551.6\,\text{slugs}} \\ &= 0.227\,\text{ft/s}^2. \end{aligned}$$

MODEL EXAM

If needed, use: $g = 10\,\text{m/s}^2 = 32\,\text{ft/s}^2$.

1. See Output Skills K1 and K2 in this module's *ID Sheet*. One or both of these skills, or neither, may be on the actual exam.
2. A socket wrench set has a weight of 16 pounds on the earth. What is its mass on the moon, where the acceleration of gravity is only 1/7 that of the earth?
3. Two high-quality toy cars having almost no friction are pushed apart by a spring. Both cars start from rest and interact only with each other via the spring and with the board on which they are rolling. One car has a mass of 1 kg and travels 15 m in 10 seconds. The other car goes 10 m in the same 10 second interval. What is the mass of the second car?

Brief Answers:

1. See this module's *text*.
2. See this module's *Problem Supplement*, problem 7.
3. See this module's *Problem Supplement*, problem 8.