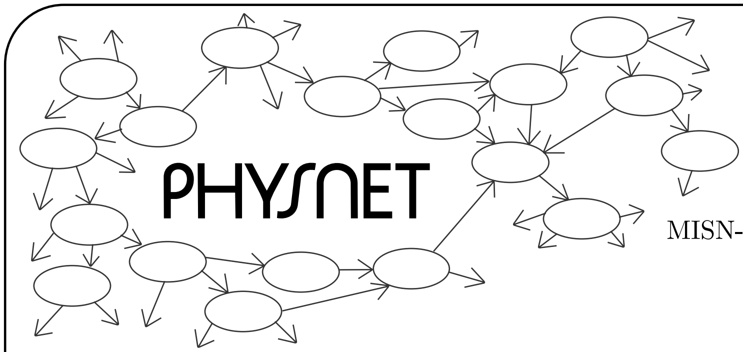
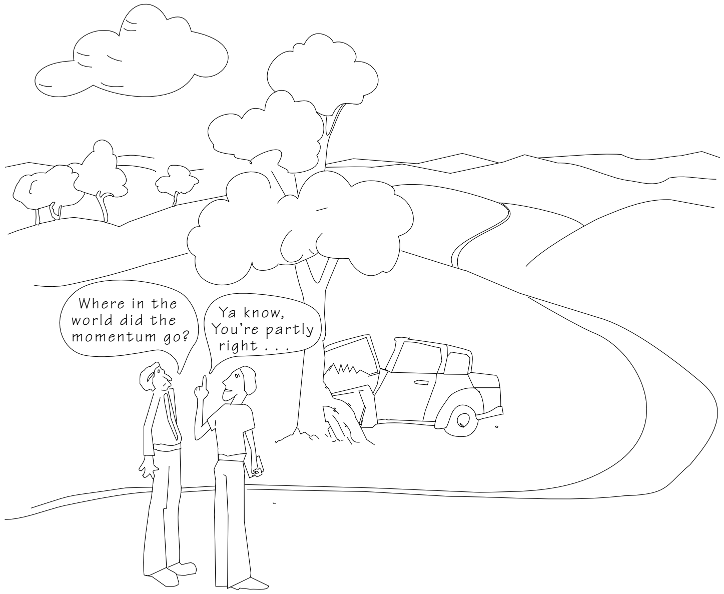


MISN-0-62

AN OVERVIEW OF PARTICLE DYNAMICS



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

AN OVERVIEW OF PARTICLE DYNAMICS
by
William Faissler, Northeastern University

Title: **An Overview of Particle Dynamics**

Author: William Faissler, Dept. of Physics, Northeastern Univ., Boston, MA

Version: 2/1/2000 Evaluation: Stage 1

Length: 1 hr; 20 pages

Input Skills:

1. A knowledge of simple kinematics (MISN-0-7).
2. A working knowledge of vectors and vector addition (MISN-0-2).

Output Skills (Knowledge):

K1. Describe the nature of a conservation law, and its use. Give an example.
K2. State the three conservation laws of particle dynamics; explain all terms and give examples.
K3. State Newton's three laws: explain the meaning of all terms, and give examples.
K4. Define momentum and kinetic energy.

Output Skills (Rule Application):

R1. Given an object whose velocity changes in a well-defined situation, determine where the object's increment of momentum and/or kinetic energy came from or went to.

Post-Options:

1. "Particle Dynamics - The Laws of Motion" (MISN-0-14).
2. "Momentum: Conservation and Transfer" (MISN-0-15).
3. "Applications of Newton's Second Law, Frictional Forces" (MISN-0-16).

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AN OVERVIEW OF PARTICLE DYNAMICS

by

William Faissler, Northeastern University

1. Introduction

1a. Why We Study Particle Dynamics. People have always found motion and its causes to be an exciting and useful subject of study. Much of man's efforts are devoted to moving things from one place to another. Why it is easier to transport a very heavy object on a boat rather than on a cart is a question of much practical interest. Certainly the natural phenomena involving motion, such as the clouds, winds, waves, tides, rivers, avalanches, etc., have always been of greatest interest to man. Even as early as Aristotle (350 B.C.) attempts were being made to formulate the rules which would explain these things. Thus interest in the subject matter of particle dynamics dates at least as far back as we have written records and continues to this day.

1b. What Particle Dynamics Is. Particle dynamics is the study of the how and why of motion that does not involve rotations. Its goal is to find a common set of rules that can explain all the varied phenomena concerning motion seen in the world around us. It aims to explain such things as why some objects fall when released while others such as helium balloons rise, why most moving objects very rapidly slow down and stop while a hockey puck on ice seems to go so far, why billiard balls bounce the way they do; why you can hit a golf ball further than you can hit a baseball, how you can make a better drag racer, why a small defensive back has so much trouble stopping a big fullback, why it is so hard to jump off a small boat, and so on. Particle dynamics also deals with such problems as why the earth does not fall into the sun, what must be done to send a rocket to the moon, and how fast a clock will run. In short, the concepts of particle dynamics can be used to explain many of the phenomena that you see in the world around you.

1c. Validity of the Laws of Particle Dynamics. The laws of physics, including those presented in this module, are justified by their utility and accuracy when used to explain a large collection of natural phenomena. They are not "proven" in the sense that theorems in geometry are proven. Neither are they "proven" by single experiments. The laws are simply very shrewd guesses of rules that might explain phenomena that are observed. Their "proof" is the fact that all of the laws and definitions taken together make up a system that enables one to predict what will happen in an experimental system. Literally millions of such experiments have been done, both very simple ones and exceedingly complex ones, and within its range of applicability, particle dynamics yields the correct answer. Based on this, we "believe" the laws—that is, we are willing to keep using them for situations in which we know they are applicable.

1d. Real Objects Are Rarely Particles. Baseballs, beer bottles and airplanes are not particles and hence cannot be treated completely by the dynamics developed in this unit. For such objects, which have extended dimensions and internal structure, we often need to extend the dynamics to include "rotational" quantities, such as angular momentum, torque, and moment of inertia.

2. Conservation Laws

2a. What a Conservation Law Is. A conservation law is nothing more than a statement that for a given system there exists some quantity which will remain constant in time, provided certain conditions are met. As an example, imagine a self-sufficient group of people living and working in an environment somewhat isolated from the rest of society, such as a secluded South Pacific Island. These people have familiar homes, jobs, businesses, etc. Some time ago they adopted American currency as the medium of exchange. Since they have no mints and since no one foolishly destroys money, the total number of dollars on the islands is fixed except when an American merchant ship makes an infrequent call to buy some rare native shells. The captain of the ship always pays for the shells with American currency, thereby increasing the total number of dollars available to the people. Between visits of the ship the total number of dollars in the society is fixed. Any money added to one person's pocket is always balanced by a decrease in the money in another's pocket. The important point here is that if at any instant the dollar possession of all the inhabitants is totaled, the sum is a constant (until a visiting ship adds new money). A physicist might describe this situation by stating a Conservation of Dollars Law: "The total number of dollars on the island is constant as long as the island is isolated (the ship is not in port)." The number of dollars any person possesses may change fifty times per day, but at any instant the sum of all island dollars is fixed, or conserved. This is an example of a conservation law.

This conservation law can be used to describe systems smaller than the whole island. For instance, if there are three people on the island who trade only with each other during some time period, then the number of dollars they have among themselves is a constant (conserved) over that time period. In this way, the law of conservation of dollars can be applied to any group that is isolated from the rest of society (not trading with the rest of society).[1]

2b. Conservation of Mass Law. Perhaps the simplest conservation law in Newtonian physics is that of mass conservation. The mass of an object is an intrinsic property of the object which, loosely speaking, is a measure of how much matter it contains. The mass of an object is independent of where the object is, how it is moving, its temperature, etc. One simple way to determine an object's mass is to weigh it (on a scale that compares it to standard weights, such as on a doctor's scale) and then compute the mass from the formula:

$$\text{mass} \;=\; \text{constant} \;\times\; \text{weight}\;,$$

(i.e., $m = cw$). If the weight is measured in pounds and mass in kilograms, then $c = 0.454\,\text{kg/lb}$. In essence, the conservation of mass law is this: If a system of masses is isolated from all other systems by excluding a flow of mass into or out of it, then the sum of the masses of the isolated system is constant. This is the case even though the number of objects may be changing (two lead blocks may be melted and solidified into one, a block of cheese may be cut into ten slices, etc.) and entities within the system may be changing mass by mass transfer between themselves (water evaporating from a glass, frictional wear of rubber from a tire, etc.).

2c. Conservation of Momentum. The total linear momentum of any system of particles is a constant provided the system is isolated from interactions with external objects. This total momentum of the system, $\vec{P}$, is calculated by adding vectorially the momenta of the constituent particles. A particle's momentum $\vec{p}$ is defined as the product of its mass and velocity: $\vec{p} = m\vec{v}$.

Figures 1-7 show a computer representation of an actual experiment that illustrates the conservation of momentum law. Figure 1 shows the paths (trajectories) of two particles of unequal masses which were isolated from all other objects and which were passing near each other. The position of each particle is shown at a number of different times. Notice that the particles were deflected from their original trajectories by some sort of interaction between the two. Figure 2 shows the velocity vector of each of the particles at the various times and Fig. 3 shows the momentum vector of each particle at each time. Just as you would add up the total number of dollars in our island example so also you find the total linear momentum of each object, using vector addition. For the two objects in our example, this summation is easy. Figure 3 also shows the total linear momentum of the system at the various times and you can see that it is constant. To summarize, for an isolated system, the vector sum of the momenta of the individual particles is a constant even though the momenta of individual particles may be changing erratically because of the interactions between the particles.

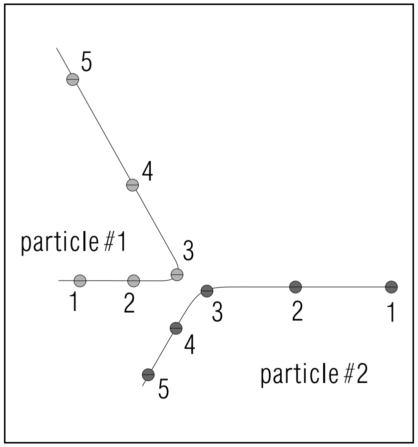

Figure 1. The paths (trajectories) of two particles that interact with each other. The numbers and marks on the trajectories indicate each particle's position at five equally-spaced instants of time.

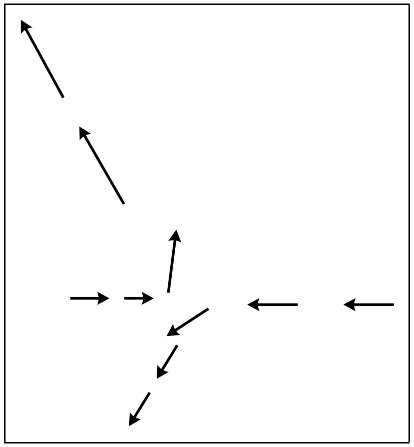

Figure 2. The velocity vectors of the two particles at the five successive times shown in Figure 1. The vectors are drawn with their tails positioned at the successive positions of the particles.

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[1] Try listing some of the conservation laws that apply in a card game - there are many, some of which are actually useful.

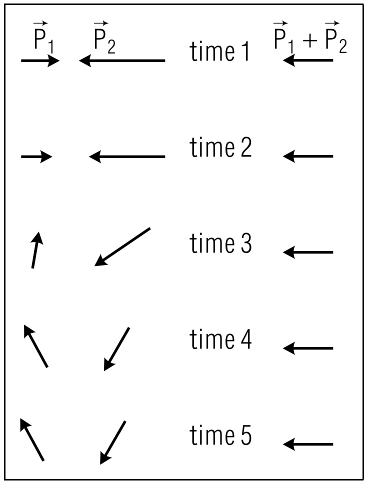

Figure 3. The momentum vectors of the two particles and the total linear momentum of the system at each of the five successive times.

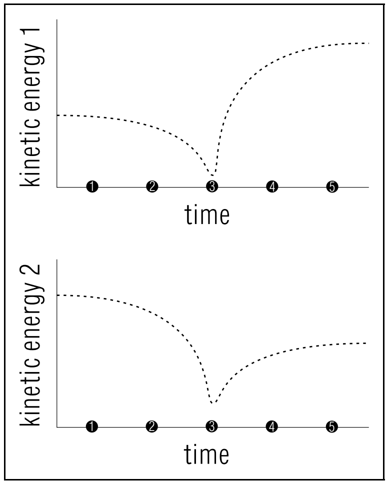

Figure 4. The kinetic energies of the two particles of Fig. 1, plotted as a function of time during their interaction.

2d. Kinetic Energy: Precise Definition. The kinetic energy of an object, labeled E_k by us, is the quantity defined as: half the object's mass times the object's velocity squared:

$$E_k = \frac{1}{2}mv^2. \tag{1}$$

Kinetic energy is a scalar quantity; it has only a magnitude and can never be negative since mass is always positive. The momentum for particle 1 is in the left column, for particle 2 in the middle and the total momentum is on the right. Note that if two objects have the same velocity, then the more massive has the greater kinetic energy. Note also that if two objects have the same kinetic energy, the less massive has the greater velocity. With a little more work you can show that if two objects have the same magnitude of momentum, mv, then the lighter object has the greater kinetic energy.

2e. Energy is Conserved. The total energy of an isolated system of objects is another conserved quantity. Figure 4 shows the kinetic energy of the particles in Figs. 1-3 moving by each other as a function of time. Figure 5 shows the total kinetic energy of the two particles as a function of time and it is quite obvious that this is not constant. However, a potential energy can be defined that depends only on the relative positions of the two particles and the nature of the interaction between them. The nature of potential energy is that, when two particles interact, their potential energy increases as their total kinetic energy correspondingly decreases. The total mechanical energy of the system consists of the sum of the kinetic energies and the potential energy. Figure 6 shows the potential energy of the system as a function of time and Fig. 7 shows the total mechanical energy, a constant. For most problems involving no friction, only those two forms of energy are needed for the conservation law. For other problems additional forms of energy must be considered; thermal energy, chemical energy, nuclear energy, radiant energy, etc. When all forms of energy are considered, it is always found that the total energy of an isolated system is a constant.

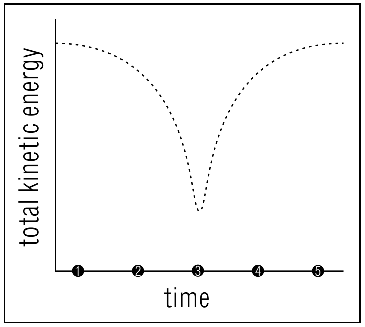

Figure 5. The total kinetic energy of the two particles as a function of time.

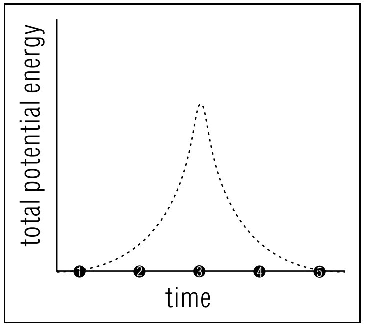

Figure 6. The potential energy of the two interacting particles as a function of time.

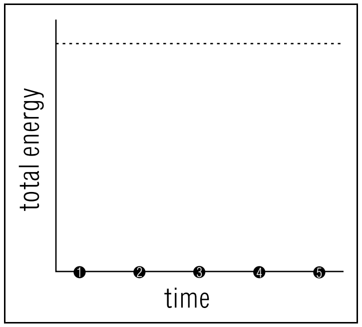

Figure 7. The total energy of the two interacting particles as a function of time. A constant!

2f. Summary. The three conservation laws that have been discussed here have many features in common and a few important differences. Each law refers to a system quantity that is a constant when the system is isolated, just as the number of dollars on the island is a constant when the island is isolated. Two of the quantities, mass and energy, are scalars while the other, momentum, is a vector. Two of the total quantities, mass and momentum, are obtained by summing the masses and momenta of the individual particles making up the system. This is not the case for the total energy. The total energy is a sum of the total kinetic energy and the potential energy. The total kinetic energy is found by summing the kinetic energies of the individual particles. However, the potential energy relates to the interaction of two or more particles, so it is not meaningful to refer to a potential energy for any individual particle but rather to the potential energy of the system as a whole.

3. The Laws of Dynamics

3a. Introduction. The laws of dynamics are the rules that tell when and how the energy and momentum of a given object may or may not change. In a similar manner, the laws of economics are the rules that tell us when and how the amount of money possessed by any one person in our island example may or may not change. When an object interacts with something else, its kinetic energy and momentum may change; the laws of dynamics tell how to calculate these effects in terms of the forces acting on the object.

3b. Other Dynamics. Newtonian particle dynamics, plus its extension with “rotational” quantities, is called “Newtonian dynamics.” Combined with Newtonian kinematics it is called “Newtonian mechanics.” Newtonian mechanics is itself a subset of “classical mechanics,” which goes beyond Newtonian dynamics to also include “special relativity.” This means it includes non-Newtonian effects that become significant when objects travel at speeds which are not slow compared to the speed of light. For dealing with accelerated frames of reference at the same time as high speeds, one must go to the higher level “theory of general relativity.” Finally, for dynamics on the atomic scale, one must use the even higher level (more general, more inclusive, wider) “quantum mechanics.”

Although the problems we customarily solve using Newtonian particle dynamics could be solved more accurately using higher level theories, the increased accuracy one would obtain is seldom significant for the problems at hand. Furthermore, solutions in those theories can only be obtained with a great deal more work if at all.

3c. Newton’s First Law. Newton’s first law says that a totally isolated object, one with no force acting on it, moves at a constant velocity. Such an object is located far from everything else, or else is so carefully located that there are no forces acting on it.

Another law, the law of momentum conservation, says that if we have a totally isolated system consisting of one object, then that object’s momentum, $m\vec{v}$, is a constant and hence its velocity is a constant. Thus it appears that Newton’s first law may just be a restatement of the law of conservation of momentum. This is not the case. All kinematic measurements must be made from some system of reference. The laws of Newtonian dynamics are found to hold only in certain kinds of systems of reference, namely, nonaccelerated ones. Newton’s first law gives us a test to apply to a measuring system to determine whether it is a suitable system in which to use Newtonian dynamics: observe an isolated object from your reference system; if it has a constant vector velocity, then Newtonian dynamics will hold in your reference system.

3d. Newton’s Second Law: How to Change Momentum. Newton’s second law says that the vector sum of all the external forces on an object equals the mass of the object times its acceleration. As an equation, this is:

$$\vec{F} = m\vec{a},$$

where one must remember that the relevant force is the net external force exerted on the object which is accelerating. Since momentum is mass times velocity and since the rate of change of velocity is its acceleration, this can be stated as:

$$\vec{F} = d\vec{p}/dt. \tag{2}$$

In words, the sum of the external forces on an object is equal to the time rate of change of the object’s momentum. This law permits you to calculate the change in the momentum of an object that is not isolated; that is, one which has forces acting on it. As such, it complements the law of momentum conservation. Note, however, that when the sum of the external forces on an object is zero, we get:

$$d\vec{p}/dt = 0 \qquad (F = 0).$$

This means that the momentum is constant, just as Newton’s first law and the law of momentum conservation say for this case. The second law also provides a method for measuring forces. A single force can be applied to an object of known mass, the acceleration can be measured, and then the force can be calculated. The force scale arising from this procedure is fully consistent with physiological experience: what you consider to be a harder push is measured as a larger force by this procedure.

3e. Newton's Third Law: Paired Reciprocal Forces. Newton's third law states that when two objects interact with each other, the force the first exerts on the second is always equal and opposite to the force the second exerts on the first. If you push on a wall, the wall pushes back on you equally hard and in exactly the opposite direction. If you push harder than the wall can push back, then part of the force you exert will go into accelerating your hand through the wall rather than being part of the force exerted on the wall. As the wall crumbles, the force it exerts on your hand drops dramatically and hence so does the force your hand exerts on the wall. Rapid acceleration of your hand results. We can use this law to show that if two objects interact only with each other, then the change in momentum of the first is equal and opposite to the change in momentum of the second. We put Newton's second law, Eq. (2), into Newton's third law for each particle:

$$\vec{F}_{1\text{ on }2} = -\vec{F}_{2\text{ on }1}$$

$$\frac{d\vec{p}_2}{dt} = -\frac{d\vec{p}_1}{dt}.$$

Integrating over some time interval,

$$\Delta\vec{p}_2 = -\Delta\vec{p}_1.$$

The total momentum change of the two-object system is just the sum of the two momentum changes:

$$\begin{aligned}\Delta\vec{P}_{\text{total}} &= \Delta\vec{p}_1 + \Delta\vec{p}_2\\ &= \Delta\vec{p}_1 - \Delta\vec{p}_1\\ &= 0.\end{aligned}$$

The total momentum is unchanged, just as the law of momentum conservation says. Thus the second law tells how a force causes momentum to change while the third law ensures that momentum will be conserved if two particles interact only with each other.

3f. How An Object's Energies Change. You have seen how a force causes an object's momentum to change. Since momentum is a vector quantity, either its direction or its magnitude or both can change. A non-zero component of force perpendicular to the velocity causes the velocity's direction to change while a non-zero parallel force component causes the magnitude of the object's velocity to change and hence it causes the object's kinetic energy to change. This parallel component also causes the object's potential energy to change. Thus, if the object's total energy stays constant, a non-zero parallel component causes a continual shift of the object's energy between potential and kinetic.

3g. Summary. The laws of dynamics can be used in situations where the forces on an individual object are known and where the conservation laws cannot be usefully used. The first law provides a way of testing a reference system to see if it is a suitable system to use. The second law relates forces to changes in an individual object's energy and momentum while the third law relates forces between objects so that momentum is conserved.

4. Examples of Using Dynamics

4a. A Small Boat Recoils. If you are standing in a small lightweight boat several feet from a float and if you attempt to jump onto the float you will most likely end up in the water. The problem is that the boat moves relatively freely about on the surface of the water (at least at low speeds). Thus you and the boat form an essentially isolated system. This system starts with zero total momentum so the total momentum will always remain zero. This means that as you gain momentum toward the float, the boat gains momentum away from it. Normally when you jump, your legs push against the earth; as you gain momentum in one direction, the earth gains momentum in the other direction, but since the mass of the earth is so much larger than your mass (about 10^{23} times) the earth gains essentially no energy. The muscles in your legs must provide only the energy needed to give you a sufficient velocity to jump the desired distance. During the course of your life, you have learned how much work your legs must do (how hard you must jump) in order to go a certain distance.

Since the mass of the boat is not all that much greater (if any) than your mass, as you jump one way the boat recoils the other way and carries off a substantial amount of kinetic energy which must be provided by your leg muscles. As a result the amount of work that would normally be sufficient to get you to the float is not enough to accelerate the boat and to get you to the, float. Hence, splash. Presumably after much experience you could get to the point where you could jump successfully from the boat to the float, but you will always find it harder than jumping an equivalent distance on land.

4b. The Universe Recoils. The results of your jumping may be far greater than you imagine. Consider that when you jump upwards, you gain a certain amount of momentum upwards. The earth must gain the same amount of momentum downwards so that the total momentum of you and the earth remains constant. However, consider that in the solar system these all form one system. According to the law of momentum conservation, as you jump one way, the whole solar system recoils in the opposite direction. Moreover, the solar system is not isolated. The solar system forms a part of our galaxy which in turn forms a part of the universe. This is really the whole system. A somewhat sobering thought is that both you and the smallest ant can cause the rest of the universe to recoil.

4c. A Train Stops. It is often the case that when considering an object that is being accelerated or decelerated, you will find that the object gains momentum from one source and energy from another. Consider an electric powered train that is slowing down. The usual technique for slowing is to turn off the motors (letting them coast) and to apply the brakes. This means that the wheels exert a force on the ground and, according to the third law, the ground exerts a force on the wheels. The latter tends to slow down the train. The momentum of the train decreases, hence the momentum of the earth increases. However, since the mass of the earth is so much greater than the mass of the train, the energy of the earth does not increase nearly as much as the kinetic energy of the train decreases. Where does the extra energy go? In fact, the energy is turned into thermal energy and used to: heat up the brakes, the track, and the air.

This is not the only way that the train can be stopped. Another way is to use the motors in reverse, running them as generators and feeding electrical energy back into the system. In this way, as the train does work on the generator the train's kinetic energy decreases. At the same time the earth still exerts a force on the wheels which tends to slow down the train. Thus the train stops and the momentum of the train is absorbed by the earth while the kinetic energy of the train is fed back into the electrical power system to be used somewhere else. Clearly both methods satisfy the laws of conservation of energy and momentum (otherwise they simply could not be), while the second method saves electrical energy and ultimately saves fossil fuel—currently a very important idea.

Acknowledgments

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PROBLEM SUPPLEMENT

Problem 4 also occurs in this module's *Model Exam*.

1. You are standing on a skateboard that is not moving. You jump up in the air, thus acquiring kinetic energy and an upward momentum. What object's momentum changed oppositely? Where did your kinetic energy come from?
2. The earth goes around the sun at the rate of 1 rev/yr. The momentum of the earth at any one time is opposite to what it was six months earlier. What object's momentum changed oppositely?
3. A car brakes to a stop at a traffic light, thus losing its momentum and kinetic energy. Where did they go?
4. A bird flying due North suddenly shifts direction to due East and slows to half its former speed. What changed momentum oppositely? Where did three-fourths of the bird's kinetic energy go?

Brief Answers:

Note: Replace each letter by the next letter in the alphabet.

1. dzqsg; ltrbkd
2. rtm
3. dzqsg; aqzjd gdzs
4. zhq; zhq jhmdshb dmdqfx

MODEL EXAM

1. See Output Skills K1-K4 in this module's *ID Sheet*.
2. A bird flying due North suddenly shifts direction to due East and slows to half its former speed. What changed momentum oppositely? Where did three-fourths of the bird's kinetic energy go?

Brief Answers:

1. See this module's *text*.
2. See this module's *Problem Supplement*, problem 4.