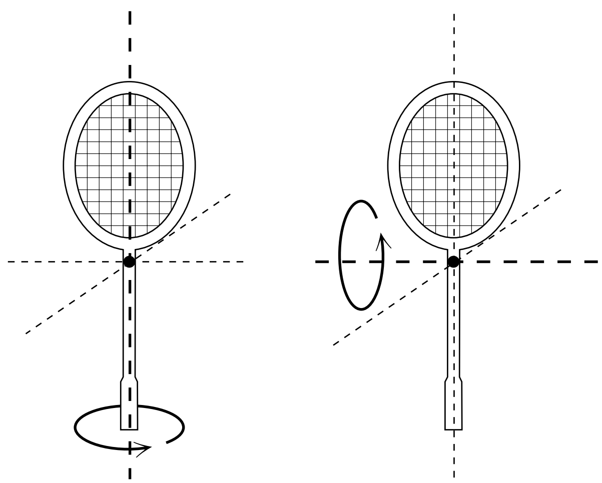
MISN-0-57

EULER'S EQUATIONS: THE TENNIS RACKET THEOREM



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

EULER'S EQUATIONS: THE TENNIS RACKET THEOREM

by

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Title: **Euler's Equations: The Tennis Racket Theorem**

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Length: 1 hr; 8 pages

Input Skills:

1. Transform the Laws of Motion from inertial to rotating systems and vice versa (MISN-0-18).
2. Describe the equations of motion for a rotating rigid body (MISN-0-36).
3. Find the principal axes of a rigid body (MISN-0-35).

Output Skills (Knowledge):

K1. Derive Euler's Equations of Motion.

K2. Use Euler's equations to prove the Tennis Racket Theorem.

Output Skills (Project):

P1. Physically demonstrate the Tennis Racket Theorem with a suitable object, explaining why two motions are stable but the other motion is not stable.

External Resources (Required):

1. V. D. Barger and M. G. Olsson, *Classical Mechanics: A Modern Perspective*, McGraw-Hill, New York (1973). For availability, see this module's *Local Guide*.

THIS IS A DEVELOPMENTAL-STAGE PUBLICATION
OF PROJECT PHYSNET

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Our publications are designed: (i) to be updated quickly in response to field tests and new scientific developments; (ii) to be used in both classroom and professional settings; (iii) to show the prerequisite dependencies existing among the various chunks of physics knowledge and skill, as a guide both to mental organization and to use of the materials; and (iv) to be adapted quickly to specific user needs ranging from single-skill instruction to complete custom textbooks.

New authors, reviewers and field testers are welcome.

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1. Introduction

For a spinning rigid body, the moments of inertia about the space-frame axes are constantly changing with time as the object rotates. In order to simplify the description of the motion, one can choose instead a coordinate system that rotates with the body. In the resulting Euler's Equations of Motion, the moments of inertia are then time-independent. An interesting application of Euler's Equations is the derivation of the surprising Tennis Racket Theorem. This theorem applies to the case where the moments of inertia about the principal axes are spaced: $I_1 \ll I_2 \ll I_3$. The theorem states that rotations about axes #l and #3 are much more stable than about axis #2, even though I_2 may be very close to I_3 in value. The theorem can be demonstrated with a tennis racket or a book (but first put a rubber band around the latter!).

2. Study Material

Study *Classical Mechanics: A Modern Perspective*, V. D. Barger and M. R. Olsson, McGraw-Hill, New York (1973), pp. 223-4 and 226-32. For availability, see this module's *Local Guide*.Equation (6-80) was discussed in MISN-0-18. All discussion of the inertia tensor or the "products of inertia" can be disregarded for present purposes. Euler is pronounced "oiler." For the "dot" notation in Equations (6-83), $\dot{\omega}$ is read "omega dot": you can discover its meaning by leafing through the book until you find the first dot over a symbol. This is a universal notation in physics. The introduction of $\tilde{\omega}$ in equation (6-108) is solely for the purpose of putting equations (6-105) and (6-106) into a form where the solutions, (6-112), are obvious. If you don't like that procedure, you can substitute (6-112) into (6-105) and (6-106) and show directly that they are solutions.

LOCAL GUIDE

The readings for this unit are on reserve for you in the Physics-Astronomy Library, Room 230 in the Physics-Astronomy Building. Ask for them as "The readings for CBI Unit 57." Do **not** ask for them by book title.

MODEL EXAM

1. See Output Skills K1-K3 in this module's *ID Sheet*. You may prove the Tennis Racket Theorem in the planar approximation, provided you prove $I_1 + I_2 = I_3$ for that case.

2. Physically demonstrate the Tennis Racket Theorem with a suitable object, explaining why two motions are stable but the other motion is not stable. You must bring the object that you use to demonstrate the effect.