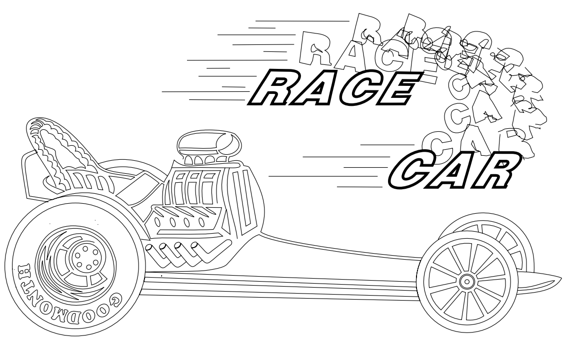
MISN-0-56

STATIC AND SLIDING FRICTION; DRAG RACER DESIGN



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

STATIC AND SLIDING FRICTION; DRAG RACER DESIGN

by

Peter Signell

Title: **Static And Sliding Friction; Drag Racer Design**

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Input Skills:

1. Given an extended body, determine its center of mass (MISN-0-6).
2. State the rules for static equilibrium (MISN-0-65).
3. Analyze frictional and gravitational forces on an object by constructing a free-body diagram. Apply Newton's laws to find the object's acceleration (MISN-0-6).
4. Given the various forces on an object moving at constant speed, compare the power needed to maintain it at that speed in (MISN-0-6).

Output Skills (Knowledge):

K1. Describe the microscopic origins of static and sliding friction, including the causes of energy loss.

K2. Derive the drag racer "CM-placement" specification for producing maximum acceleration.

K3. Derive the Huntington-Fox MPH law in the Constant Power approximation: $\text{MPH} = K \times (\text{Power}/\text{Weight})^{1/3}$ and evaluate K_{theory}.

External Resources (Optional):

1. V. D. Barger and M. G. Olsson, *Classical Mechanics: A Modern Perspective*, McGraw-Hill, New York (1973). For availability, see this module's *Local Guide*.

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New authors, reviewers and field testers are welcome.

STATIC AND SLIDING FRICTION; DRAG RACER DESIGN

by

Peter Signell

1. Introduction

Static and sliding friction are essential parts of everyday life: without the first you could not even walk. What is the origin of friction? How can a normal force produce a force at right angles to it, the frictional force? These questions are tackled directly, and the First Law of Friction ($f = \mu N$) is used to optimize a drag racer design.

2. The Readings

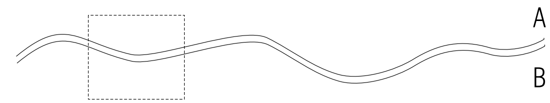
2a. Comments on the Readings. Rita Livingston's letter and Geoff Fox's reply are a discussion of what is known about the basic mechanisms of friction and wear. Arnold Aron's comment on Geoff Fox's reply constitutes an attempt to clarify some of the language and mental images used in this area. Geoff Fox's discussion of drag racing pulls together many aspects of mechanics, including friction, to get at some aspects of drag racer design. For a discussion of the drag racer in a textbook see *Classical Mechanics: A Modern Perspective*, V. D.Barger and M G. Olsson, McGraw-Hill Book Co., N.Y. (1973), pp.6-8. For availability, see this module's *Local Guide*.

2b. "Does Weight Have a Horizontal Component?" by Rita Livingstone. Reprinted by permission from *The Physics Teacher*, **11**, 288 (1973).

When my students first asked this question, my immediate response was "No Way!" Then they countered with "How can weight influence friction on a horizontal surface?"

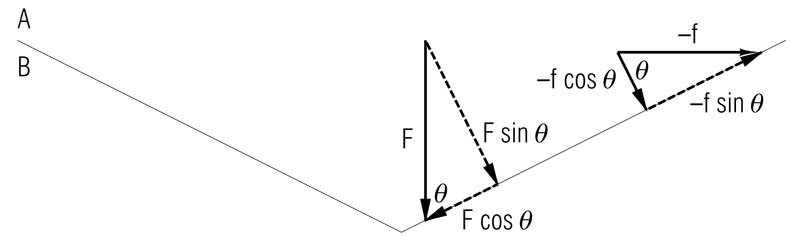
The formula which is used to calculate friction $\vec{f}$ in terms of the normal force $\vec{F}$ is $\vec{f} = \mu\vec{F}$. Since weight is the normal force on a horizontal surface, we do have the result that weight does influence the horizontal force of friction.

What follows is my suggestion of a partial explanation. I'd like to expose it to some critical review by other teachers.



Above we see surface A resting on surface B much magnified so that the irregularities of the surface are visible. We shall assume that irregularities of A fit into those of B and that they assume that position when A rests on B.

Below we have greatly magnified the region in the dotted box.



The weight of A is shown by the vector $\vec{F}$ and $\vec{F}$ is resolved into its components along the irregularity in A and perpendicular to the irregularity. If θ is the angle between the vertical and the slope of the irregularity then the components are $\vec{F}\cos\theta$ and $\vec{F}\sin\theta$ as shown in the figure.

If we are to move A to the right at constant velocity, A must be raised along the incline which requires $\vec{F}\cos\theta$ be balanced by an opposite force. This force is supplied by a horizontal force $-\vec{f}$; that is, opposite but equal in magnitude to friction. This force is shown resolved into components as before.

In effect the force applied to overcome friction is shown as two components. One which is pushing A closer to B and one lifting A along the irregularity.

Since $-\vec{f}$ was only sufficient to balance $\vec{f}$ we have

$$\vec{F}\cos\theta = -\vec{f}\sin\theta$$

$$\vec{f} = -\frac{\vec{F}\cos\theta}{\sin\theta},$$

$$\vec{f} = -\vec{F}\cos\theta\,.$$

This shows that the relation $\vec{f} = \mu\vec{F}$ can be derived from these assumptions. It is worth noting that smoother surfaces would imply $\theta \rightarrow 90°$ so that $\cot\theta \rightarrow \theta$ and $\vec{f} \rightarrow 0$. That is, with smoother surfaces friction drops as we expect.

I suspect this is only a part of the story as some reading in the literature suggests the components perpendicular to the irregularity cause a bonding of materials. I'd be quite interested in hearing if researchers have any definite statements about how much friction results from the irregularities of the surfaces and how much from bonding.

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2c. A Reply by Geoffrey Fox. Reprinted by permission from *The Physics Teacher*, **11**, 288 (1973).

As [Ms.] Livingston remarked, weight does not have a horizontal component. Yet, as the students replied, weight does influence friction on a horizontal surface. Indeed, one of the "Laws of Friction" states that the frictional force is always proportional to the normal force. Weight happens to be the normal force in this particular case. The real puzzler, then, seems to be "By what mechanism does the normal force influence the frictional force?" This is by no means a trivial question. It is a question that has been with us for a very long time. And yet it is a question for which a simple comprehensive answer has not yet been given. The "roughness hypothesis," a simple version of which Mrs. Livingston has presented, was first presented by Coulomb. Until the middle of this century it was widely accepted. It has recently[1] been show to be unsatisfactory, however. The theory of friction accepted today is called the "adhesion hypothesis." It suggests that there are microscopic regions of the surfaces that "weld" together and it is the breaking of these bonds which causes friction.

With regard to the "roughness hypothesis" as presented here, there is a very basic difficulty. Friction, it must be remembered, is a non-conservative force. It represents a dissipation of energy. All attempts at building a model of the mechanism of friction that involve only conservative forces are thus doomed to failure. For example, there is no reason to believe that part A is always being lifted, which takes energy, and never lowered, which releases energy. On the average, of course, these

[1]Frederic Palmer, Am. J. Phys. **17**, 181; 327 (1949).

two processes balance one another out assuming the final vertical position of object A is the same as its initial vertical position.

A similar argument can be developed against the "adhesion hypothesis," i.e., for every "weld" that is broken there is a weld that is formed and thus there is no net dissipation of energy. To explain the dissipation of energy in the "adhesion hypothesis," we must look more closely into what happens when a weld is broken. The situation is analogous to that of stretching a spring until it breaks. When the spring breaks, both portions of the spring will be oscillating. The energy involved in breaking the spring/weld has gone into vibrational energy. Because the welded region is connected to the entire solid, however, its vibrational energy diffuses throughout the entire solid. Thus in the final state the entire solid possesses more vibrational energy; i.e., it is at a higher temperature; it has been heated up. When a new weld is formed, the region near the weld does not possess an excess of vibrational energy and the process is clearly non-symmetrical. The dissipation of energy is thus explained.

A similar argument could be made for the "roughness hypothesis." The vertical motions of the body or regions of the body could produce local vibrations that diffuse throughout the body heating it up. However, measurements of the actual vertical motions indicate they are not large enough to explain the frictional forces involved.[2] It is interesting to note that for either of these two mechanisms to be operative, the solid must be nonrigid. If a solid were truly rigid, then there would be no internal vibrations and energy could not be dissipated in this manner.

There are two other related mechanisms for friction that are important. One of these is the phenomenon of wear. In wear small portions of the objects are broken off in an irreversible manner. The binding energy of the particle worn off thus accounts for a frictional force. Other permanent deformations of the solids are possible, such as the plowing of one surface by the other. Since permanent deformations are inherently irreversible they too can account for frictional forces.

The most interesting aspect about sliding friction is that the definitive paper on the subject has not yet been written. There is still much interesting research that is waiting to be done on this seemingly pedestrian topic.

[2]C. D. Strang and C. R. Lewis, J. Appl. Phys. **20**, 1164 (1949).

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2d. "Friction - There's the Rub!" by Arnold Arons. Reprinted by Permission from *The Physics Teacher*, **11**, 453 (1973).

Professor Geoffrey Fox's response in the "Stumpers" column to Ms. Rita Livingston's query about the relation between weight of a sliding object and the horizontal frictional force[3] is entirely correct, but I would like to amplify two aspects of the situation that are rarely pointed out to students with sufficient emphasis and clarity. These aspects play a crucial role in understanding the concept of "friction."

(1) I suggest that it is inadvisable either for us or the students to use locutions such as "weight is the normal force on a horizontal surface." We give the name "weight" to the gravitational force the earth exerts on the block. The block, when placed on the horizontal surface, exerts a force on the surface, and I urge that this always be referred to as the "normal force" and described as "the force exerted by the block on the surface." (An equal and opposite force is exerted by the surface on the block. This force appears only on the force diagram of the block, while the previously mentioned force appears only on the force diagram of the surface.) It is the normal force that measures the effect "squeezing" the two surfaces together.

The normal force is not, in general, equal to the weight of the block. This is why we should avoid the locution "weight is the normal force on a horizontal surface." If we bear down on the block, the normal force has a greater magnitude than the weight of the block. If we tug the block upward without lifting it off the surface, the normal force is smaller than the weight of the block. The normal force is not identical with the weight of the block; it is a different force in its own right, and it only occasionally happens to be numerically equal to the weight of the block. This fundamental distinction between the weight of the block and the normal force must be clarified for the students if they are to comprehend further aspects of the problem. The concept should not be allowed to become confused through careless use of language.

[3]R. Livingston and G. Fox, Phys. Teach. **11**, 288 (1973).

It is also useful to say something about how the normal force originates. This is a "passive" force that develops through an elastic, spring-like effect as the table, or other supporting structure, sags or compresses - as an easy chair sags when we sit in it. The sag, just like the stretch of the spring balance, adjusts itself to the total force acting from above, i.e., to the normal force, which is not necessarily equal to the weight of the block. The sag is always there in the presence of a normal force, even if it is only a piece of paper that is placed on the table and the effect is too small to be visible to the naked eye. If the load is increased indefinitely, the table eventually breaks or the supporting surface gives in some other way. The system cannot adjust itself to keep increasing the passive force indefinitely.

(2) Now consider the horizontal force f that we describe as "frictional." It is only in special circumstances that f happens to have the particular value μN, and this fact should be brought out explicitly for the students.

Suppose we are not applying any external horizontal force to the block. Under these conditions $f = 0$; were this not the case, the block would accelerate horizontally. How would it know which way to go?

As we begin to push, very gently, horizontally on the block, the acceleration remains zero. We are forced to conclude that f increases from zero and adjusts itself so that it exactly balances the external force we are applying. In other words, f is a passive force that increases just as N does when we bear down on the block and is sustained through the "welding" mechanism described by Prof. Fox. The presence of the force f is accompanied by elastic shear deformation of the block and the supporting surface just as the presence of the normal force N is accompanied by elastic bending or compression. Just like N, the frictional force f assumes an infinity of values, depending on the externally applied force - values from zero on up to the point at which something "gives." In the case of friction, it is the "welds" that give and the block begins to slide.

It is only at this special condition, when the block begins to slide, that $f_{\max} = \mu N$, where μ is the parameter we usually call the "coefficient of static friction." The statement $f_{\max} = \mu N$ is simply a semi-empirical recognition that the largest value of the frictional force sustained by the particular pair of surfaces depends not only on the surfaces themselves but also on the the normal force squeezing them together. Rather than revealing a depth of knowledge, both the name "friction" and the over-simplification represented by μN serve to conceal our ignorance most pervasive physical phenomena manifest in the world around us.

It is a shame that, apparently because of the excesses embodied in a previous generation of textbooks, virtually all mention of friction has been eliminated (a new excess) from some current texts. Perhaps the next generation will return to a development embodying at least the modest phenomenological aspects discussed in the May "Stumpers" column and my present amplification. Friction is far too obvious, familiar, and important a phenomenon to be cavalierly ignored; even Galileo did not "wish it away" to the degree many of our courses now imply.

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2d. "On the Physics of Drag Racing" by Geoffrey Fox. Reprinted by Permission from *The American Journal of Physics*, **41**, 311 (1973).

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The sport of drag racing can be very usefully examined in an introductory mechanics course. The concepts of energy, momentum, acceleration, velocity, torque, and power are all involved in the analysis of the problem. Many students find the problem to be very contemporary and close to their own experience. High interest among the students usually results in increased understanding of the physical principles involved.

INTRODUCTION

As undergraduate physics majors my brother, Robert C. Fox, Jr., and I spent our leisure time building and racing vehicles in drag racing competition. We found our knowledge of physics very useful in that realm, enabling us to understand relationships involved in building and driving a winning car. Entering graduate school, I was forced financially to discontinue this colorful sport. Instead, I contented myself with continuation of theoretical studies and compilation of those made previously. As an assistant professor of physics, I have given seminars on the subject which have been extremely well received. I have also used particular topics as special

credit problems in both my introductory physics course for engineers and scientists and in an upper division course in intermediate mechanics. Especially students in mechanics have been stimulated by the consideration of this topic.

I. WHAT IS A DRAG RACE?

Drag racing is an extremely popular spectator/participant sport in the United States which is rapidly growing. Interest among persons in their teens and early twenties is particularly high. If one omits horse racing, which I believe is popular only because of the gambling ingredient, then automobile racing is the largest spectator sport in the country. A drag race is the simplest of all forms of auto racing. The course is straight and therefore the problems are essentially one- or at most two-dimensional. The race is an acceleration contest from a standing start over a distance of $\frac{1}{4}$ mile. Two variables are measured for each competitor, they are ET and MPH. ET stands for elapsed time and is just the time it takes from crossing the starting line until crossing the finish line. MPH is essentially the terminal speed at the end of the 1320 ft, measured through a 132 ft time trap on either side of the finish line. Both measurements are done electronically with the aid of photocells.

The motor vehicles that compete in drag racing are many and varied. They range from everyday street-driven vehicles which have ET's of 20 sec and MPH of 70, through hopped up street legal cars turning 120 mph in 11.00 sec on up to the all out dragsters running over 220 mph in slightly more than 6 sec.

II. AN EXAMINATION OF THE VARIABLES

In trying to develop any theory it is always wise to start from an examination of the variables. In drag racing there are really just two dependent variables, ET and MPH. The goal of our theory is to predict or explain these dependent variables as a function of the independent variables. The key difficulty is that the list of independent variables is quite long, some of the key ones are listed: vehicle weight, engine horsepower (also the details of the horsepower curve), location of the center of gravity (both horizontal and vertical), coefficient of friction of tires on road (this in turn is a function of tire size, tire pressure, normal force, and surface conditions to mention a few), air drag (depends basically on frontal area and drag coefficient), gearing (both rear axle and transmission), shifting techniques, moment of inertia of rotating parts, driver skill, etc.

III. THE FIRST ORDER THEORY OR THE CONSTANT POWER APPROXIMATION (CPA)

In trying to develop a first order theory it would be useful to examine the basic rules which determine the breakdown of classes in drag racing.

The key parameter here is the ratio of weight to engine displacement. Engine displacement, is of course, closely related to power, hence our first order theory could be expected to use power to weight ratio as the independent variable. In the early 1960's an engineer, Roger Huntington, developed an empirical law relating power to weight ratio to MPH. His rule is $MPH = K_{\rm exp}(\,\text{Power}/\,\text{Weight})^{1/3}$. The empirical $K_{\rm exp}$ is 225 where power is in horsepower, weight in pounds, and MPH in miles per hour.[4] No such simple relationship has been found for ET however.

Huntington's empirical rule has a theoretical basis which I discovered in 1964. It is just the result to be expected in the Constant Power Approximation (CPA). This is easily developed in the following manner,

$$\Delta\text{Energy} = \text{Work}$$

$$\frac{1}{2}mv^2 = \int_0^t Pdt.$$

Assuming constant power one gets

$$\frac{1}{2}mv^2 = Pt$$

or:

$$v = (2Pt/m)^{1/2}\,.$$

Since

$$x = \int_0^t vdt = \frac{2}{3}\left(\frac{2P}{m}\right)^{1/2} t^{2/3}$$

we get

$$t = [(3/2)(m/2P)^{1/2}x]^{2/3}\,.$$

Now we eliminate time from Eqs. (1) and (2) giving

$$v = (3xP/m)^{1/3}$$

which is of the same form as the empirical law except $K_{\rm theory} = 270$, using the same units as previously.[5]

Although the discrepancy between $K_{\rm theory}$ and $K_{\rm exp}$ doesn't appear to be large, if one cubes it one finds that about 50% of the theoretical

[4]Private communication.

[5]PHYSNET note: We find $K_{\rm theory} = 281\,\text{mph}(\,\text{lb}/\text{hp})^{1/3}$. Note that, as is common in engineering practice, the proportionality constant $K_{\rm theory}$ includes units conversion factors.

power is wasted. How can one account for this? There are a number of factors which can account for power loss. Some are: power losses in the drive train, shifting losses (no power is transmitted during shifting), average engine horsepower is less than peak engine horsepower (in the rpm range used), wind and rolling resistance, full power cannot be transmitted initially due to traction limitations (traction), effective weight (due to Moment of Inertia and Rotational Energy).

Of these the last two are the most important. Consider the last first. Energy $= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}M_{\rm eff}v^2$ and $\frac{1}{2}M_{\rm eff} = M + I(\omega/v)^2$. And we must consider I due to both those items rotating at axle speed and also those rotating at engine speed. The latter includes clutch, flywheel, crankshaft, etc. Notice that ω/v which is related to gearing comes in as a squared term, thus for a typical vehicle like our own 1955 Chevy 339 in^3 (total weight 3700 lb) the $I(\omega/v)$ term accounts for the following additional weight in each gear: 1st gear, 1500 lb; 2nd gear, 800 lb; 3rd gear, 500 lb; 4th gear, 250 lb.

Obviously this is a very important correction factor, but still there is the question: "Why does the first order theory work so well?" Or phased another way, "Why doesn't MPH depend very strongly on traction whereas ET is extremely sensitive to this effect?" To answer this question let us proceed to the second order theory.

IV. THE SECOND ORDER THEORY

In this theory we divide up the run into two portions: the traction limited portion, and the power limited portion. For the traction limited portion of the run we have

$$F = const. = \mu_e M g$$

in which μ_e is related to the coefficient of friction through geometrical factors and includes "weight transfer" effects. In practice μ_e varies between 0.5 and 2.0. Thus ...

Acknowledgments

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LOCAL GUIDE

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