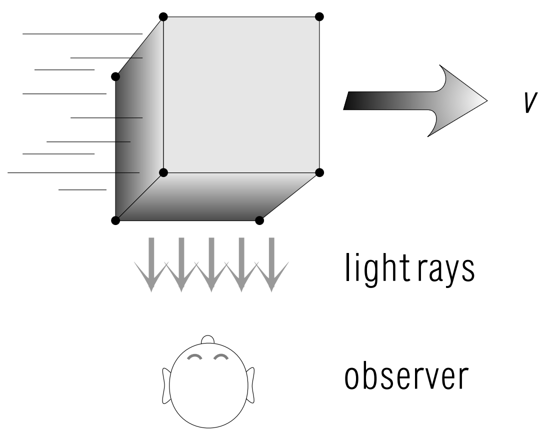
MISN-0-44

APPEARANCES AT RELATIVISTIC SPEEDS



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

APPEARANCES AT RELATIVISTIC SPEEDS

by

Peter Signell, Michigan State University

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Title: **Appearances at Relativistic Speeds**

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Input Skills:

1. Use relativistically correct equations to determine line segment lengths as observed from different frames of reference (MISN-0-13).

Output Skills (Knowledge):

K1. Derive the correct appearance of a moving object, taking into account the apparent Lorentz contraction of the object and also the finite speed of the particles of light by which one observes it.

K2. Show that, for ordinary every-day speeds, the apparent angle of rotation, produced by retardation/contraction, is too small to be seen.

K3. Describe the appearance of a moving cube as its $v/c \to 1$.

K4. Show that, under that assumption of the Lorentz contraction alone, a moving object would appear distorted with respect to its rest-frame shape.

K5. Show that a distant straight line traveling at speed v will appear rotated through an angle $\theta = \cos^{-1}\sqrt{1 - v^2/c^2}$.

APPEARANCES AT RELATIVISTIC SPEEDS

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1. Replacing an Erroneous View

This topic, the appearance of objects moving at relativistic speeds, should be viewed as a recreation inasmuch as its final result is not used elsewhere in the physical sciences. Nevertheless, the derivation and exposition in 1959 led to widespread discussion; in one stroke it demolished a picture which physicists had believed in and taught for fifty years. Moreover, it replaced a complicated erroneous picture with a simple correct one. We also recommend the derivation to you because the techniques and principles used are useful in other areas.

2. Simple Object: a Cube

In order to simplify the derivation, we will deal solely with an object which, in its rest frame (when at rest), has the appearance of a cube with sides of length ℓ. The cube is assumed to be so far away from you, the observer, that the light rays coming to you from its various parts can be considered to be parallel. Also, we will only consider its appearance as the object passes directly in front of you as shown in Figure 1. Note also

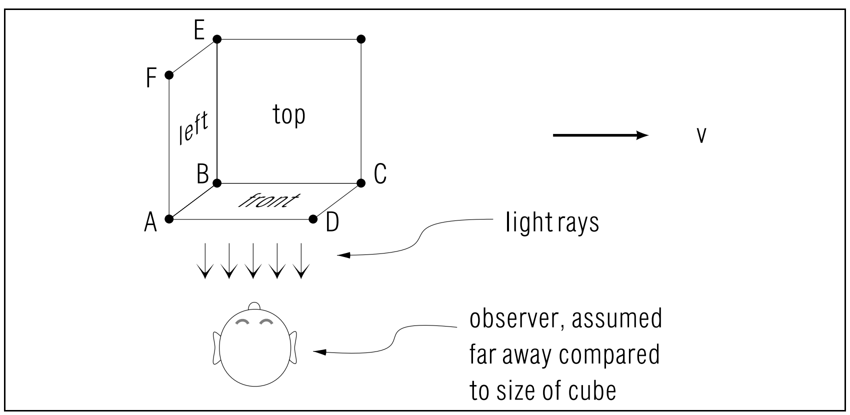

Figure 1. View from above. The cube is drawn as it would appear in its rest frame.

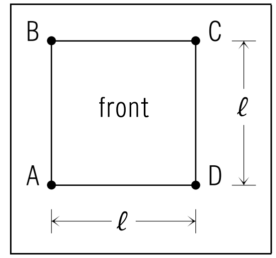

Figure 2. As in Fig. 1, view from the front. What corner is hidden behind corner B?

the labels of the outside corners of this solid cube.

3. No Contraction, No Retardation

3a. View From Side of Slowly Moving Cube. This is the ordinary approximate appearance one obtains for an object moving at speeds that are very small compared to the speed of light. For a cube of side ℓ we expect the appearance to be as shown in Figure 2.

3b. View of a Slowly Moving Rotated Cube. Now suppose that the cube has been rotated about an axis perpendicular to both the line to the eye and the direction of motion, as shown in Figure 3a. Then the observer is predicted to see the appearance as shown in Figure 3b.

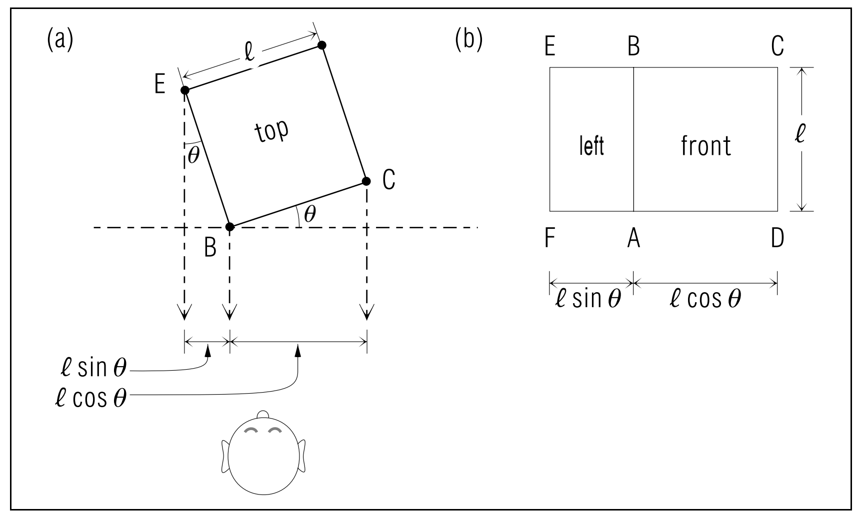

Figure 3. A rotated cube: (a) top view; (b) side view.

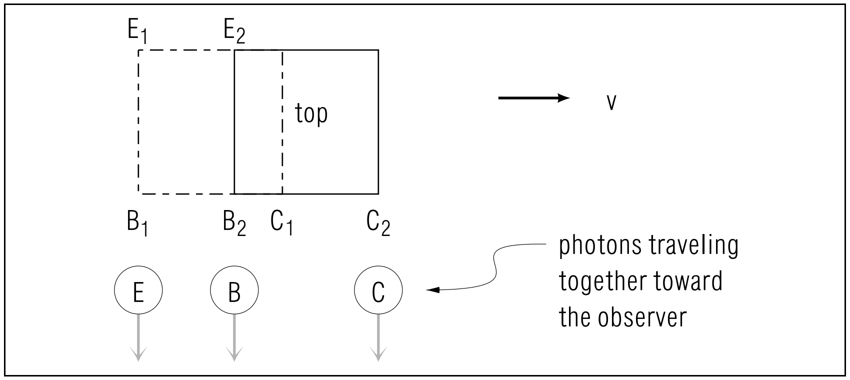

Figure 4. View of the cube from above. The dashed lines show the position of the cube at some particular time while the solid lines show its position at a later time.

4. Retardation, No Contraction

4a. Photon Departure Positions and Times. We go back to the unrotated cube and start from the appearance at any one instant as resulting from all of the photons[1] arriving at the observer at that instant. Photons do not travel instantaneously from the cube to the observer. The speed of light is finite. Due to the varying distances of the parts of the cube from the observer, and our requirement that the photons from them must all arrive at the eye at the same time, the photons must have left their points of origin on the cube at various times in the past. This "retardation" effect means that a photon of light must have left corner E earlier than one from corner B to arrive at the observer at the same time. To see this, compare Figure 1 and Figure 4. In Figure 4, photon E must have left corner E at time t_E, whereas photons B and C left at time $t_{B,C}$.

4b. A Photon's Lead Time. The extra distance the E photon must travel is ℓ, since that is the length of side BE (remember that our photons are supposed to be traveling parallel paths due to the large distance to the observer). Thus the E photon must start out earlier than the B and C photons by a time given by:

$$E\text{-photon's lead time} \;=\; \frac{\text{extra distance}}{\text{photon's speed}} = \frac{\ell}{c} = t_{B,C} - t_E. \text{(see Fig. 4)}$$

[1](fō′tän, a particle of light).

4c. Distance Traveled by Cube During Lead Time. During this time the cube travels a distance given by:

$$\text{cube's distance} \;=\; \text{cube's speed} \;\times\; \text{time} = v \times \frac{\ell}{c} = \frac{\ell v}{c} = t_{B,C} - t_E.$$

Considering only the retardation effect, the cube would appear as in Fig. 5.

5. Retardation and Contraction

5a. Modification Due to Lorentz Contraction. Using relativity,[2] one finds that a moving length in the direction of motion becomes contracted from its "rest-frame" or proper value L_0 to $L(v) = L_0\sqrt{1 - v^2/c^2}$. Side AD of the cube in Fig. 1 has its entire length in the direction of motion. Therefore the Lorentz contraction modifies Fig. 5 in the manner shown in Fig. 6.

5b. Apparent Angle of Rotation. This is the final, correct appearance. It is astonishing that it is exactly the rest-frame appearance of the cube rotated through an angle of:

$$\theta = \sin^{-1}(v/c) = \cos^{-1}\sqrt{1 - v^2/c^2}\,,$$

as shown in Figure 3b.

▷ Show that Figs. 5 and 6 reduce to Figure 2 as $v/c \to 0$.

▷ Show that, for ordinary everyday speeds, θ is too small to be seen.

▷ Describe the appearance of the cube as $v/c \to 1$.

▷ Prior to Terrell's inclusion of the effects of retardation, physicists assumed that the appearance would be that produced by the Lorentz contraction alone. Show, under that assumption, that the front side (toward

[2]See "The Length Contraction and Time Dilation Effects of Special Relativity" (MISN-0-13).

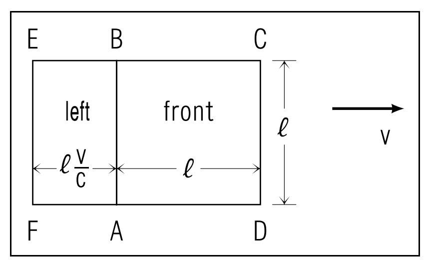

Figure 5. From the side, no Lorentz contraction but with retardation.

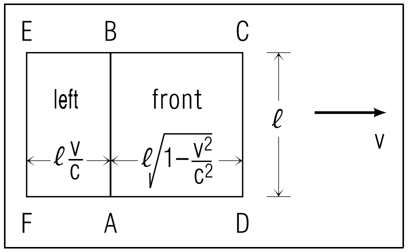

Figure 6. Appearance from the side, with Lorentz contraction and retardation.

the observer) (in the direction of motion) would appear rotated through an angle $\theta = \cos^{-1}\sqrt{1 - v^2/c^2}$, while the left side (perpendicular to the direction of motion) would appear unrotated. You are thus showing that the cube would appear distorted with respect to its rest-frame shape.

▷ Suppose a distant piece of straight line has its length in the direction of its motion. Show that it will have the appearance of a rest-frame line rotated through an angle $\theta = \cos^{-1}\sqrt{1 - v^2/c^2}$.

Acknowledgments

Steve Smith, Mark McChesney, Douglas Ullmer, and Ray Van Ausdal gave valuable feedback on an earlier version of this module. Jim Linnemann made very useful suggestions that were incorporated as given. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

A. Additional References

- "Invisibility of the Lorentz Contraction," James Terrell, *Physical Review 116*, 1041 (1959). See the first page of his article, reproduced by permission in Appendix B.
- "Observation of Length by a Single Observer," Roy Weinstein, *American Journal of Physics 28*, 607 (1960). He considers the problem of a line segment oriented along the direction of motion, with Lorentz contraction and retardation, and without the restriction of parallel rays (large distance from object to observer). See his abstract, reproduced by permission in Appendix B.
- "The Visual Appearance of Rapidly Moving Objects," V. F. Weisskopf, *Physics Today*, Vol. 13, No. 9, 24 (Sept., 1960). The de-

scription in this module is based mainly on Professor Weisskopf's exposition of Terrell's paper. Weisskopf shows the appearance of the cube along its entire trajectory and makes interesting comments.

- *Scientific American*, Vol. 203, No. 1, 74 (1960).
- "Apparent Shape of Large Objects at Relativistic Speeds", Mary L. Boas, *American Journal of Physics 29*, 283 (1961). She considers objects of finite shape and distance. Her abstract is reproduced by permission in Appendix B.

B. Journal Excerpts

Invisibility of the Lorentz Contraction*

JAMES TERRELL
Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico
(Received June 22, 1959)

It is shown that, if the apparent directions of objects are plotted as points on a sphere surrounding the observer, the Lorentz transformation corresponds to a conformal transformation on the surface of this sphere. Thus, for sufficiently small subtended solid angle, an object will appear—optically—the same shape to all observers. A sphere will photograph with precisely the same circular outline whether stationary or in motion with respect to the camera. An object of less symmetry than a sphere, such as a meter stick, will appear, when in rapid motion with respect to an observer, to have undergone rotation, not contraction. The extent of this rotation is given by the aberration angle $(\theta-\theta')$, in which θ is the angle at which the object is seen by the observer and θ' is the angle at which the object would be seen by another observer at the same point stationary with respect to the object. Observers photographing the meter stick simultaneously from the same position will obtain precisely the same picture, except for a change in scale given by the Doppler shift ratio, irrespective of their velocity relative to the meter stick. Even if methods of measuring distance, such as stereoscopic photography, are used, the Lorentz contraction will not be visible, although correction for the finite velocity of light will reveal it to be present.

INTRODUCTION

EVER since Einstein presented his special theory of relativity[1] in 1905 there seems to have been a general belief that the Lorentz contraction should be visible to the eye. Indeed, Lorentz stated[2] in 1922 that the contraction could be photographed. Similar statements appear in other references too numerous to be mentioned, and even Einstein's first paper leaves the impression,[3] perhaps unintentionally, that the contraction due to relativistic motion should be visible. The usual statement is that moving objects "appear contracted," which is somewhat ambiguous. The special theory predicts that the contraction can be observed by a suitable experiment, and the words "observe" and "see" seem to be used interchangeably in this connection.

There is, however, a clear distinction between observing and seeing. An observation of the shape of a fast-moving object involves simultaneous measurement of the position of a number of points on the object. If done by means of light, all the quanta should leave the surface simultaneously, as determined in the observer's system, but will arrive at the observer's position at different times. Similar restrictions would apply to the use of radar as an observational method. In such observations the data received must be corrected for the finite velocity of light, using measured distances to various points of the moving object. In seeing the object, on the other hand, or photographing it, all the light quanta arrive simultaneously at the eye (or shutter), having departed from the object at various earlier times. Clearly this should make a difference between the contracted shape which is in principle observable and the actual visual appearance of a fast-moving object.

CONFORMALITY OF ABERRATION

The basic question of the visibility of the Lorentz contraction may be stated as that of the appearance of a rapidly moving object in an instantaneous photograph. The object, of known shape when at rest, is assumed to have a high uniform speed relative to the camera. The camera is assumed to be at rest in a Galilean (unaccelerated) frame of reference. Of course it would make no difference if the camera were, instead, considered to move at high speed past the stationary object, but the photograph produced must be examined at rest, so it is simpler to consider the camera as stationary. The mechanism of the camera must be such as to give it essentially instantaneous shutter speed and sharp focus over the necessary depth of field.

The questions of whether to use photographic film which lies in a plane or is curved so that all points are at the same distance from the lens (or pinhole), and whether to use a lens corrected to eliminate optical distortions, could be troublesome. To simplify matters, it is assumed that the object subtends a visual solid angle sufficiently small that these matters need not be considered. It is assumed that the camera is pointed directly at the apparent position of the object, so that the light rays strike the film in a perpendicular direction, producing an image in the center of the photographic film. The camera is assumed, also for simplicity,

* This work was supported by the U. S. Atomic Energy Commission.

1 A. Einstein, Ann. Physik 17, 891 (1905).

2 H. A. Lorentz, *Lectures on Theoretical Physics* (Macmillan and Company, Ltd., London, 1931; translated from Dutch edition of 1922), Vol. 3, p. 203.

3 In reference 1 [English translation from *The Principle of Relativity* (Dover Publications, Inc., New York, reprinted from 1923 Methuen edition)] Einstein stated: "A rigid body which, measured in a state of rest, has the form of a sphere, therefore has in a state of motion—viewed [betrachtet] from the stationary system—the form of an ellipsoid of revolution with the axes $R(1-v^2/c^2)^{\frac{1}{2}}$, R,R. Thus, whereas the Y and Z dimensions of the sphere (and therefore of every rigid body of no matter what form) do not appear [nicht erscheinen] modified by the motion, the X dimension appears [erscheint] shortened in the ratio $1:(1-v^2/c^2)^{\frac{1}{2}}$, i.e., the greater the value of v, the greater the shortening. For $v=c$ all moving objects—viewed [betrachtet] from the "stationary" system—shrivel up into plane figures."

AMERICAN JOURNAL of PHYSICS
A Journal Devoted to the Instructional and Cultural Aspects of Physical Science
VOLUME 28, NUMBER 7 OCTOBER, 1960

Observation of Length by a Single Observer*

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(Received December 3, 1959)

One problem arising in teaching special relativity is the confusion in many texts of the thought experiments, used in developing the theory, with other simple laboratory operations. As an example we consider here the observation of length. The existence of the Lorentz-Fitzgerald contraction has led educators to conclude that one *sees* a contraction of a rapidly moving body. However, the act of *seeing* involves a single observer, while the observation of the Lorentz-Fitzgerald contraction requires at least two observers. It is shown here that the length seen by a single observer is not the usual contraction, and indeed, under certain circumstances, one sees a body considerably lengthened rather than contracted.

AMERICAN JOURNAL of PHYSICS
A Journal Devoted to the Instructional and Cultural Aspects of Physical Science
VOLUME 29, NUMBER 5 MAY, 1961

Apparent Shape of Large Objects at Relativistic Speeds

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(Received September 21, 1960)

It has been recently recognized that there is a difference between the measured Lorentz contracted shape of an object moving at relativistic speed and the shape as seen by a single observer. The case of an object which subtends a small solid angle at the observer has been discussed by several authors. This paper discusses objects so large or so near that the subtended solid angle cannot be considered small, and gives simple proofs that spheres always present a circular outline and that straight lines may appear curved. These results are applied to revise Gamow's well-known picture of the bicyclist seen by Mr. Tompkins.

MODEL EXAM

1. See Output Skills K1-K5 in this module's *ID Sheet*. The actual exam may contain any number of these skills.

Brief Answers:

1. See this module's *text*.