

ELIMINATING OSCILLATOR TRANSIENTS: A PROJECT by
Peter Signell

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## Title: Eliminating Oscillator Transients: a Project

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## Input Skills:

1. Relate the qualitative aspects of the behavior of the displacement as a function of time of the damped driven oscillator to the quantities: initial displacement, initial velocity, damping constant, natural resonant frequency and amplitude of the sinusoidal driving force (MISN-0-31).

## Output Skills (Project):

P1. Use an interactive computer terminal and a canned program for the numerical solution of a second order inhomogeneous differential equation of physics, a solution whose desired characteristics have been specified. Explain the physics principle involved in obtaining the solution.

## External Resources (Required):

1. Access to a computer with the Numerov program.
2. Graph paper.

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Our publications are designed: (i) to be updated quickly in response to field tests and new scientific developments; (ii) to be used in both classroom and professional settings; (iii) to show the prerequisite dependencies existing among the various chunks of physics knowledge and skill, as a guide both to mental organization and to use of the materials; and (iv) to be adapted quickly to specific user needs ranging from single-skill instruction to complete custom textbooks.

New authors, reviewers and field testers are welcome.

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## 1. Introduction

1a. Real World Oscillators are Damped by Dissipative Forces.
In the real world, oscillators are always damped by dissipative forces. For example, if you start a mass oscillating up and down at the end of a vertically-hung spring, the mass will sooner or later come to rest due mainly to internal friction in the spring. This decrease of amplitude with time is called "damping": it is a damping of the oscillations. An oscillator is often driven by an oscillating force which supplies power to compensate for the frictional damping losses.

1b. The Project. In physics one sometimes uses a "canned," computer program to solve an equation or analyze data. In this project you use a canned computer program, via an interactive computer terminal, to solve the equation for the damped driven harmonic oscillator. Your specific task is to find initial conditions such that transient effects are eliminated, leaving a pure sinusoidal response.

## 1c. Computer Calculates Solution at a Finite Set of Times.

 Since the method used by the computer is a numerical one, it can only produce values for the displacement $x(t)$ at a finite set of times. We choose to calculate $x(t)$ at evenly spaced times, consecutively spaced by time increments $\Delta$. However, the approximate method used for obtaining the solution depends on $\Delta$ being small; if $\Delta$ is too large, the solution will be inaccurate.
## 2. The Computer Program "Numerov"

The computer program NUMEROV uses the Numerov algorithm to solve Newton's Second Law ${ }^{1}$ for a damped driven oscillator which has a free frequency of $\omega=\sqrt{k / m}=40 \pi \mathrm{~s}^{-1}$ and an oscillator mass of $m=$ 1 kg . You specify the damping strength $\lambda$, the driving-force strength $F_{0}$, the time increment $\Delta$, the driving force period $P=2 \pi / \omega$, and the

[^0]displacement $x_{0}$ and velocity $v_{0}$ at time $t=0$, all in SI units. In the PROCEDURE section we recommend specific values for $\lambda, F_{0}, \Delta, P, x_{0}$, and $v_{0}$, which have been chosen to give you an interesting looking graph. Feel free to pick other values close to those shown or to pick completely different values. For the latter case you will probably want to do some planning so that, for example, the interference between the steady-state and transient solutions is apparent on your graph.

## 3. Procedure

a. Follow the PROGRAM NUMEROV DIRECTIONS ${ }^{2}$ once, using:

$$
\lambda=0, F_{0}=0, \Delta=0.00125, P=0, x_{0}=0, v_{0}=75
$$

and a second time with the time interval made too crude;

$$
\lambda=0, F_{0}=0, \Delta=0.01, P=0, x_{0}=0, v_{0}=75
$$

Plot these two smooth curves on a sheet of graph paper, using different colors for the two cases or a solid line for one and a line of dashes for the other. On that same sheet of graph paper plot the exact functional solution for SHM with the above values for $x_{0}$ and $v_{0}$. Remember our PROGRAM NUMEROV's value of $\omega=\sqrt{k / m}=40 \pi \mathrm{~s}^{-1}$. We suggest that for this part you use a scientific calculator.
b. Follow the PROGRAM NUMEROV DIRECTIONS again, using:

$$
\lambda=23.03, F_{0}=0, \Delta=0.00125, P=0.1, x_{0}=0, v_{0}=75
$$

to find the damped undriven transient solution, and:

$$
\lambda=23.03, F_{0}=7178, \Delta=0.00125, P=0.1, x_{0}=0, v_{0}=75
$$

to find the full damped driven solution. Plot these two curves on the other sheet of graph paper.
c. Take the long-time, steady-state, purely sinusoidal part of the solution and extrapolate it back to $t=0$ in order to determine the values of $x_{0}$ and $v_{0}$ it would have had there. You can either determine these values directly from your graph or you can use the 2-point derivative formula given below, along with the computer-output numbers. No other math or methods are needed.

[^1]The two-point derivative formula referred to is:

$$
f^{\prime}(x)=\frac{f(x+\Delta)-f(x-\Delta)}{2 \Delta}
$$

d. Follow the PROGRAM NUMEROV DIRECTIONS once more, but now using your calculated steady-state values of $x_{0}$ and $v_{0}$ :

$$
\lambda=23.03, F_{0}=7178, \Delta=0.00125, P=0.1, x_{0}=?, v_{0}=?
$$

If you have done everything correctly, the transient response will be gone. Plot this curve on the graph paper, showing it to blend in with the previous solution as the latter's transient dies away.
e. Make sure that your finished graph lines are clearly labeled.
f. Extra (No Credit): You may like to show that the steady state amplitude $B$ and phase $\beta$ on your graph check the formal solutions. ${ }^{3}$ You may also like to plot the solution for the values of $\omega=\omega_{0} / 2, \omega_{0}$, and $2 \omega$ in order to see how things change as the driving force passes through the resonance frequency $\omega_{0}$.
Notes on How to Interpret Computer Output:

- ". $625-3$ " means " $0.625 \times 10^{-3}$."
- The displacement values $x(t)$ are only printed for every first, second, fourth or eighth time net points and are printed consecutively across the page starting in the second column. The first column gives the time (in SI units) for the displacement value next to it in the second column.


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Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant \#SED 74-20088 to Michigan State University.

[^2]
## LOCAL GUIDE

Go to the CBI Consulting Room during your class's regular scheduled hours and ask for a copy of "CBI document $\mathbf{8 - 1 0 0}$ " or purchase a copy at our current Copying Store (for the name of the store, see the last page of the Materials Lists section of your updated CBI Student Handbook).
Follow the directions in 8-100 for the program "NUMEROV.EXE." The program's output is dumped to a file named "NUMEROV.DAT" from which it can be printed and/or copied to a floppy disk.
When you come to take an exam that includes this unit, you must bring the originals of your computer output and graphs and staple them to the rest of your exam papers. If you submit a copy of any part, rather than an original, you will receive a score of zero for unit 30 .

Your computer output and graphs must be annotated so as to show what you did and how your graphs are to be interpreted. No credit will be given for time spent merely running a computer and producing output: the project must be done properly and the results properly interpreted.


[^0]:    ${ }^{1}$ See "The Numerov Algorithm for Problems Involving Newton's Second Law" (MISN-0-39), where this method is derived.

[^1]:    ${ }^{2}$ See this module's LOCAL GUIDE, attached.

[^2]:    ${ }^{3}$ See "Damped Driven Oscillations; Mechanical Resonances" (MISN-0-31) for the formal solutions.

