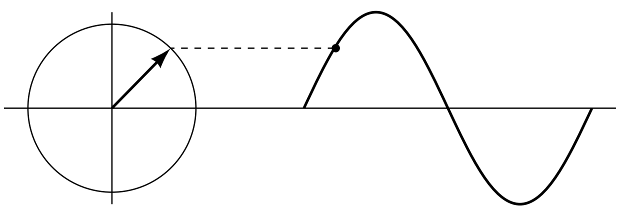
PHYSNET

MISN-0-27

PHASORS



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

PHASORS

by

Peter Signell, Michigan State University

1. Study Procedures

Title: **Phasors**

Author: Peter Signell, Michigan State University

Version: 4/25/2002 Evaluation: Stage 4

Length: 1 hr; 12 pages

Input Skills:

1. Vocabulary: acceleration, amplitude, angular frequency, displacement, restoring force, frequency, initial phase, phase (MISN-0-430) or (MISN-0-25).

Output Skills (Knowledge):

K1. Vocabulary: phasor, phasor diagram.

K2. Describe the relative positions and the motions of phasors on a phasor diagram representing the displacement, velocity and acceleration of a harmonic oscillator, and the restoring force acting on it.

K3. State how the actual displacement, velocity and acceleration of a harmonic oscillator and the actual restoring force can be determined from a phasor diagram.

Output Skills (Problem Solving):

S1. Given a harmonic oscillator with specified amplitude, frequency, and initial phase, draw a phasor diagram illustrating the displacement, velocity, and acceleration of the oscillator and the restoring force acting on it at a specific time. Describe how the diagram changes if any one of the given parameters is changed continuously from its initial value.

External Resources (Required):

1. M. Alonso and E. J. Finn, *Physics*, Addison-Wesley (1970). For availability, see this module's *Local Guide*.

PHASORS

by

Peter Signell, Michigan State University

1. Study Procedures

1a. Readings. Read Section 9.2 in AF[1] and examine Fig. 1. This figure shows the phase relationships among various quantities associated with a harmonic oscillator whose displacement is given by[2] $x(t) = A\cos(\omega t + \alpha)$, where α is the "initial phase." The rotating vectors are called "phasors" and do not, as vectors, correspond to any real quantity. That is why they are marked with asterisks. The $y*$-axis is an artificial construct and is not to be confused with an axis in a physical space. Here the x-axis is the real line of motion of the oscillator.

[1] M. Alonso and E. J. Finn, *Physics*, Addison-Wesley (1970), on reserve for you in the Physics Library as "Readings for Unit 27."

[2] The sine function could be used just as well; that substitution would have exactly the same effect as subtracting a constant ninety degrees from the cosine phase. *Help: [S-1]*

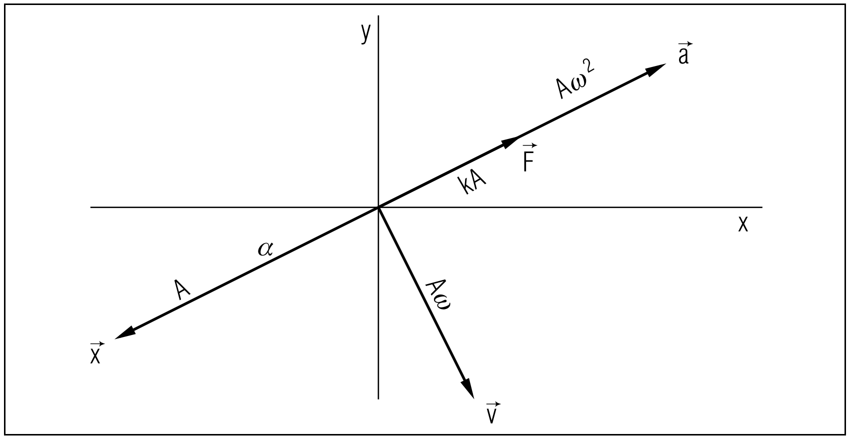


Figure 1. Phasor diagram showing the displacement, velocity, acceleration, and force phasors for a harmonic oscillator at time zero. Each phasor has a different horizontal and vertical scale (length units for $\vec{x}*$, length-per-unit-time units for $\vec{v}*$, etc.). Note that the lengths of the phasors are written on them.

1b. Answer these questions. From the readings, determine:

1. Which are real, the phasors or their projections along the x-axis?
2. What do the lengths of the phasors signify?
3. What do the directions of the phasors signify?
4. Which way do the phasors rotate, clockwise or counter-clockwise?
5. Why is the velocity phasor drawn ninety degrees "ahead" of the displacement phasor rather than ninety degrees "behind" it?
6. Are the phasors all locked together, or do some rotate faster than others?
7. At what angular velocity do the phasors rotate?
8. How can you tell from the form of $x(t)$ that the phase angle should be measured counter-clockwise, from the positive x-axis to the $\vec{x}*$-phasor?

1c. Work problems. Work the problems in the Problem Supplement.

LOCAL GUIDE

The readings for this unit are on reserve for you in the Physics-Astronomy Library, Room 230 in the Physics-Astronomy Building. Ask for them as "The readings for CBI Unit 27." Do **not** ask for them by book title.

PROBLEM SUPPLEMENT

1. A harmonic oscillator of mass 0.64 kg has a displacement from equilibrium given by:

$$x = (2.1\,\mathrm{m})\,\cos\left[(1.25/\,\mathrm{s})t + (\pi/4)\right].$$

   a. Draw a phasor diagram showing the displacement, velocity, and acceleration phasors of the oscillator, and the phasor representing the restoring force acting on it, at time $t = 0$.

   b. Draw another phasor diagram of the four phasors at time $t = 1.0\,\mathrm{s}$. Describe how the phasor diagram changes during the time interval between $t = 0$ and $t = 1.0\,\mathrm{s}$.

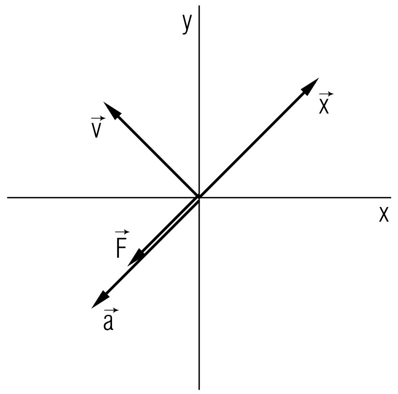
2. Draw a phasor diagram showing the displacement, velocity, acceleration and restoring force phasors at $t = P/2$ where P is period for the case:

$$x(t) = A\,\cos\left[(\pi/s)t + \pi/2)\right].$$

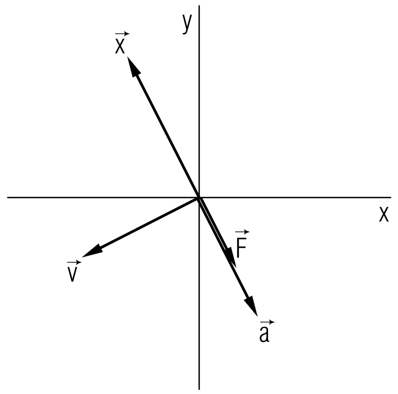
   Also, show the total phase and mark its value.

Brief Answers:

1. a. $|\vec{x}| = 1.5\,\mathrm{m}$,
   $|\vec{v}| = 1.9\,\mathrm{m/s}$,
   $|\vec{a}| = 2.3\,\mathrm{m/s^2}$,
   $|\vec{F}| = 1.5\,\mathrm{N}$.



b. The four phasors maintain their directions relative to one another, and all rotate counter-clockwise with an angular speed of 72 degrees per second.



2. The total phase is: $\omega t + \alpha = 270°$ at $t = P/2$.

SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from TX-1a)*

Either a sine or a cosine function can be use to represent the displacement of the harmonic oscillator, with an appropriate initial phase for each case, i.e.

$$x = A\ \sin(\omega t + \alpha)$$

or

$$x = A\ \cos(\omega t + \alpha')$$

where

$$\alpha' = \alpha + \frac{\pi}{2}.$$

How does this choice of a harmonic function determine which axis (the positive x-axis or the negative y-axis) is chosen as the reference for the phase angle?

MODEL EXAM

1. See Output Skills K1-K3 in this module's *ID Sheet*.
2. Draw a phasor diagram showing the displacement, velocity, acceleration and restoring force phasors at $t = P/2$ where P is period for the case:

$$x(t) = A \cos\left[(\pi/s)t + \pi/2\right].$$

   Also, show the total phase and mark its value.

Brief Answers:

1. See assigned readings.
2. See Problem 2 in the Problem Supplement.