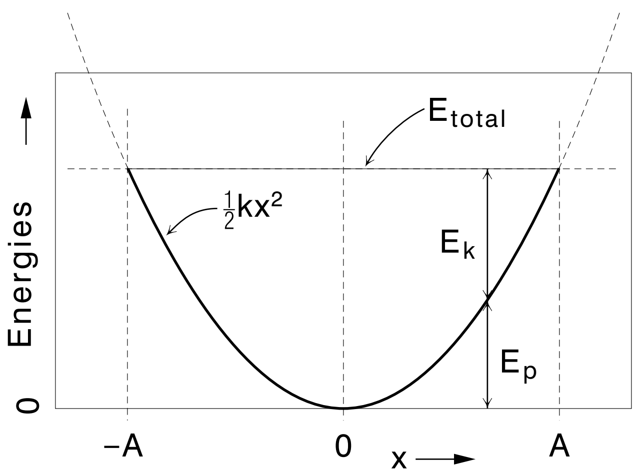
PHYSNET

MISN-0-25

SIMPLE HARMONIC MOTION



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

SIMPLE HARMONIC MOTION

by

Kirby Morgan, Charlotte, MI

Title: **Simple Harmonic Motion**

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Length: 1 hr; 24 pages

Input Skills:

1. Vocabulary: angular frequency, frequency, uniform circular motion (MISN-0-9), kinetic energy (0-20), potential energy, total energy (0-21).
2. Write down the coordinates, as a function of time, of a particle in uniform circular motion (MISN-0-9).
3. Relate the one-dimensional position of a particle, as a function of time, to the particle's velocity and acceleration, and the force acting on it (MISN-0-15).
4. Given the total energy of a particle, plus a graph of its potential energy versus its one-dimensional coordinate position, describe its motion and the force on it as time progresses (MISN-0-22).

Output Skills (Knowledge):

K1. Vocabulary: oscillatory motion, simple harmonic oscillator, simple harmonic motion, displacement, initial time, amplitude, phase, scaled phase space, frequency, period.

K2. Write down a general equation for SHM displacement as a function of time, assuming zero initial phase and maximum initial displacement, and identify the amplitude, angular frequency, phase, and displacement. Derive the corresponding equations for velocity and acceleration.

Output Skills (Problem Solving):

S1. For a specific SHO, use given items in this list to produce others, as requested: displacement, time, frequency, period, phase, velocity, angular frequency, acceleration, force, kinetic energy, potential energy, total energy, and word and graphical descriptions of the motion in real space and scaled phase space.

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New authors, reviewers and field testers are welcome.

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1. Oscillatory Motion and SHM

1a. Oscillatory Motion. One of the most important regular motions encountered in science and technology is oscillatory motion. Oscillatory (or vibrational) motion is any motion that repeats itself periodically, i.e. goes back and forth over the same path, making each complete trip or cycle in an equal interval of time. Some examples include a simple pendulum swinging back and forth and a mass moving up and down when suspended from the end of a spring (see Fig. 1). Other examples are a vibrating guitar string, air molecules in a sound wave, ionic centers in solids, and many kinds of machines.

1b. Simple Harmonic Motion. The basic kind of oscillatory motion is Simple Harmonic Motion; it is used to analyze most other oscillatory motions for purposes of understanding and design. A motion is said to be Simple Harmonic if the oscillating object's position, $x(t)$, can be represented mathematically by a "harmonic" function of time; that is, by a sine or cosine function. The word "simple" refers to a need for only one sine or cosine term to represent the position of the oscillating object. Any object that undergoes simple harmonic motion (abbreviated SHM) is called a simple harmonic oscillator (here abbreviated SHO).

1c. Uses of SHM. The mathematical techniques used in the study of simple harmonic motion form the basis for understanding many phenomena: the interactions of elementary particles, atoms, and molecules; the sounds of various musical instruments, radio and television broadcasting; high-fidelity sound reproduction, and the nature of light and color. They are used to study the vibrations in car engines, aircraft wings, and shock absorbers, and to study brain waves.

2. The Kinematics of SHM

2a. The Displacement Equation. By definition, a particle is said to be in simple harmonic motion if its displacement x from the center point

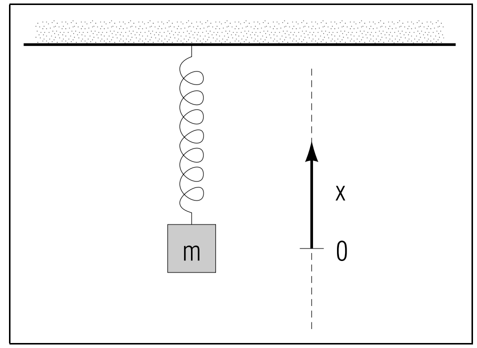


Figure 1. A weight on a spring.

of the oscillations can be expressed this way:[1]

$$x(t) = A\cos(2\pi\nu t)\ , \tag{1}$$

where ν is the frequency of the oscillation and t is the elapsed time since a time when the displacement x was equal to A.

▷ Show that $x(0) = A$, regardless of the value of ν. *Help: [S-1]*

2b. Example: Mass on Vertical Spring. We illustrate Eq. 1 with the example of an object with mass oscillating up and down at the end of a vertical spring, as in Fig. 1. The displacement x is then the height of the object, measured from the center point of the oscillations. This height needs to be at its maximum value at time zero since Eq. 1 produces $x(0) = A$, which is the maximum value the displacement can have. We can assure that this is true by measuring t on a stopwatch which we start at a time when the object is precisely at the top point of its motion. Alternatively, we can grab the mass and move it up to $x = A$, then let it go at the exact time we start the stopwatch.

▷ Show that if we had used a sine function instead of a cosine in Eq. 1, we would have had to start the stopwatch at a time when the object was passing through $x = 0$ headed upward. *Help: [S-14]*

[1] For definitions of the symbols used in this equation, see "Uniform Circular Motion," MISN-0-9. A more general form of Eq. 1, useful when you do not want to restrict the initial time, is presented in "SHM: Boundary Conditions," MISN-0-26.

2c. Displacement Equation Parameters. The cosine function varies between -1 and $+1$ so the value of the displacement from center, $x(t)$, varies between $-A$ and $+A$. The maximum displacement, A, is called the "amplitude" of the motion. Typical units are *meters*.

Typical units for the frequency are *cycles per second*, also called *Hertz*, abbreviated Hz.

The quantity $2\pi\nu t$, the *argument* of the cosine, is called the motion's "phase." Typical units are *radians* and *degrees*. Although the phase has the units of an angle, it does not usually correspond to a space angle in the problem (look at the system in Fig. 1 where obviously there is no angle involved).

The time-derivative of the phase is called the "angular frequency" and is denoted by the symbol ω:

$$\omega = 2\pi\nu\,.$$

This enables us to write Eq. (1) in a more succinct form:

$$x = A\cos(\omega t)\,. \qquad (2)$$

Typical units for ω are *radians per second*.

▷ Contrast the units of ω with those of ν. *Help: [S-15]*

▷ How would you write the displacement equation for a particle in simple harmonic motion with an angular frequency of 4π rad/s and an amplitude of 5 cm; and what is the displacement of the particle at $t = 1.0$ s? *Help: [S-2]*

2d. The Oscillator's Period. The "period" of the oscillation is defined as the amount of time it takes for the oscillator to go through one complete oscillation or "cycle." Since the cosine function repeats itself whenever ωt is increased by 2π, it repeats itself whenever t is increased by $2\pi/\omega$. This, then, is the period, T, of a SHO:

$$T = \frac{2\pi}{\omega}\;. \qquad (3)$$

The period is the inverse of the frequency; that is,

$$T = \frac{1}{\nu} = \frac{2\pi}{\omega}\;. \qquad (4)$$

▷ Find the period and frequency for an angular frequency of 4π rad/s. *Help: [S-3]*

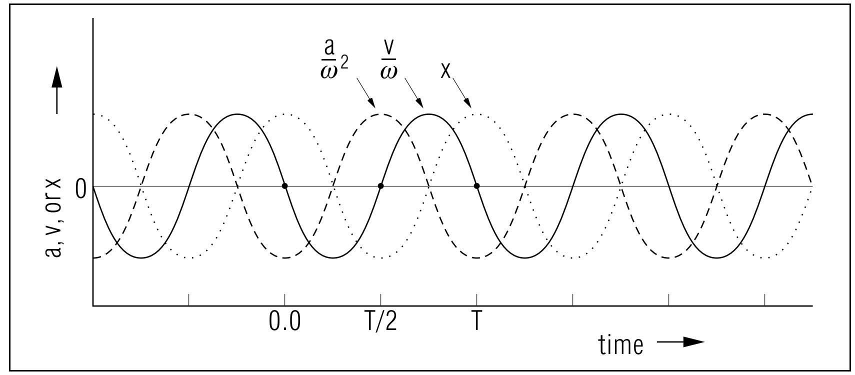


Figure 2. Displacement, velocity and acceleration versus time for an SHO. The velocity has been scaled by $(1/\omega)$ and the acceleration by $(1/\omega^2)$. The quantity T is the SHO's period.

2e. Velocity and Acceleration in SHM. The velocity and acceleration of an SHO can be easily found by differentiating the displacement equation, Eq. (2). The velocity is

$$v = \frac{dx}{dt} = -A\omega\sin(\omega t) \qquad (5)$$

and the acceleration is

$$a = \frac{dv}{dt} = -A\omega^2\cos(\omega t) \qquad (6)$$

so:

$$a = -\omega^2 x\;. \qquad (7)$$

Equation (7) shows that an SHO's acceleration is proportional and opposite to its displacement. We have plotted x, v/ω, and a/ω^2 as functions of time in Fig. 2.

▷ Make sure that you, yourself, can construct and interpret Fig. 2! *Help: [S-6]*

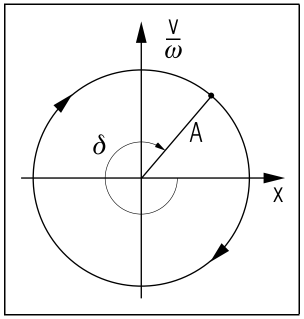


Figure 3. v/ω vs. x for the SHO in Fig. 2.

2f. SHM in a Scaled Phase Space. In developing an understanding of the time-development of SHM, it is useful to look at a plot of the oscillator's displacement versus its velocity. At any specific time, displacement and velocity each have a specific value and so determine a point on the plot of displacement vs. velocity. As time goes on, the SHO's displacement and velocity change so the corresponding point on the plot moves accordingly. Since the SHO's displacement and velocity are cyclical, the point on the plot traverses the same complete closed path once every cycle.

In order to simplify the SHO's displacement vs. velocity trajectory, we scale the velocity-axis by a factor of $1/\omega$ (see Fig. 3). Then any SHO's trajectory will be a circle of radius A (as shown in Fig. 3). In this space, the SHO's point is always at the "phase" angle, δ, marked off clockwise from the positive x-axis (see Fig. 3).

As time increases the point representing the SHO moves around the circle of radius A with constant angular velocity $-\omega$ (the minus sign merely means the motion is clockwise). Its angular position at any particular time is the phase angle at that time.

▷ Show that the circle in Fig. 3 and the clockwise motion around it follow from Eqs. (2) and (5). *Help: [S-8]*

▷ Show that $(-\omega)$ is the angular velocity of the point that simultaneously represents the oscillator's position and velocity in Fig. 3. *Help: [S-9]*

▷ For a real SHO, demonstrate v/ω vs. x as t increases, as in Fig. 3 (see [D-2] in this module's Demonstration Supplement).

3. The Dynamics of SHM

3a. Force is Proportional and Opposite to Displacement. Using Newton's second law, $F = ma$, and Eq. (7), $a = -\omega^2 x$, it is easy to find the force necessary for a particle of mass m to oscillate with simple harmonic motion:

$$F = -m\omega^2 x\,. \tag{8}$$

Note that the force on an SHO is linearly proportional to its displacement but has the opposite sign. For a positive displacement the force is negative, pointing back toward the origin. For a negative displacement, the force is positive, again pointing back toward the origin. A force which is linear and always points back to the place where F = 0 is called a "linear restoring force." For simplicity we write Eq. (8) in the form:

$$F = -kx\,. \tag{9}$$

where $k \equiv m\omega^2$ is called the "force constant" (or "spring constant" or "spring stiffness") for the particular oscillator being observed.

▷ Write down ω, T, and ν in terms of k and m for SHM. For a weight-on-spring SHO, show that as its mass is increased its spring must be stiffened in order that its amplitude and frequency remain unchanged. *Help: [S-7]*

▷ Sketch a plot of F versus x for an SHO. On your plot, identify k. *Help: [S-10]*

▷ Describe the motion of an SHO's point on a plot of F vs. x. Contrast the motion of the point on this plot with that of the similar point in Fig. 3. *Help: [S-5]*

▷ For a real SHO, demonstrate F vs. x as t increases (see [D-3] in this module's Demonstration Supplement).

3b. Potential and Kinetic Energy for SHM. Knowing the force acting on an SHO, we can calculate its kinetic and potential energy.

First, to obtain the potential energy we use $F = -kx$ and the definition of potential energy:[2]

$$E_p = -\int F dx = \int kx dx = \frac{1}{2}kx^2\,. \tag{10}$$

Thus the potential energy has its minimum value, zero, at $x = 0$.

[2]See "Potential Energy, Conservative Forces, The Law of Conservation of Energy" (MISN-0-21).

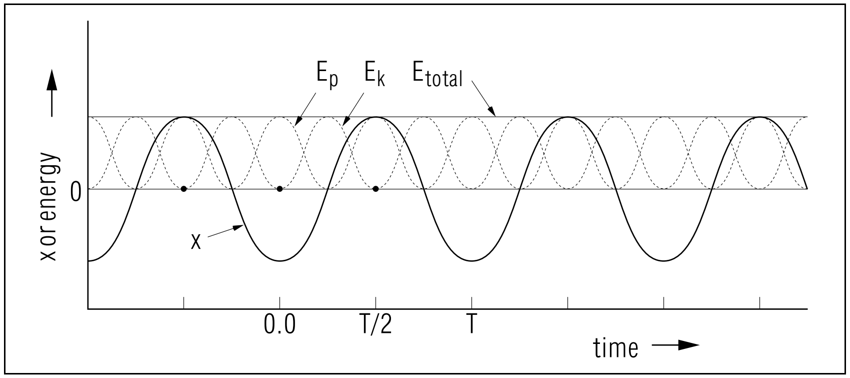


Figure 4. Kinetic, potential, and total energy and displacement vs. time for the SHO of Fig. 2.

The kinetic energy of the SHO is:

$$
\begin{aligned}
E_k &= \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t) = \frac{1}{2}m\omega^2[A^2 - A^2\cos^2(\omega t)] \\
&= \frac{1}{2}k(A^2 - x^2)\,. \qquad (11)
\end{aligned}
$$

The kinetic energy is a maximum at $x = 0$ and is zero at the extremes of the motion ($x = \pm A$), as shown in Fig. 4.

3c. Total Energy for SHM. The total energy of a simple harmonic oscillator is:

$$E = E_k + E_p = \frac{1}{2}kA^2\,, \qquad (12)$$

which is a constant quantity (independent of time). Thus during an oscillation, as the potential energy increases and decreases, the kinetic energy decreases and increases so the total energy remains constant.

The potential energy curve, $E_p = k\,x^2/2$ (which is a parabola), and the total energy curve $E = k\,A^2/2$ (which is a horizontal line), are shown as functions of displacement in Fig. 5. The points where the line and the curve intersect ($x = \pm A$) are the limits of the motion.

▷ Use Fig. 5 to describe the major events in the values of the potential, kinetic, and total energies, and the displacement, as time advances during a complete SHM cycle. *Help: [S-4]*

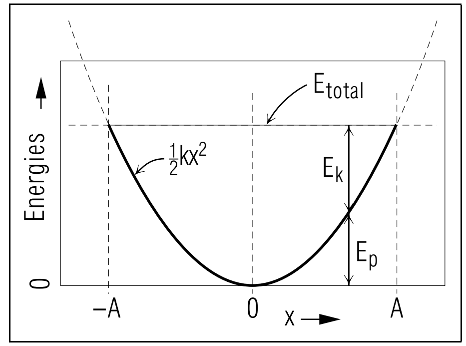


Figure 5. Potential and total energy vs. displacement for an SHO.

▷ Use words alone (no graph) to describe the major events in the values of the potential, kinetic, and total energies, and the displacement, as time advances during a complete SHM cycle. *Help: [S-11]*

Acknowledgments

Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

Glossary

- **amplitude**: maximum value of displacement.
- **angular frequency**: time rate of change of the phase.
- **angular velocity in SHM**: the angular velocity of the SHO's point in scaled phase space. Its value is the negative of the SHO's angular frequency ω.
- **displacement**: position relative to the center-point of the SHM.
- **frequency**: number of complete cycles per unit time.
- **harmonic function**: a sine or cosine function.
- **oscillatory motion**: motion that exactly repeats itself periodically.
- **period**: the time for one complete cycle.

- **phase**: the argument of the harmonic function describing the SHM. Here we have chosen the initial time to be when the displacement is at a maximum, so the harmonic function is a cosine and its phase angle δ is ωt.
- **scaled phase space**: a space in which the two axes are the SHO's displacement x and v/ω. The current state of an SHO is a point in this space. The point continually traverses a circle of radius A with constant angular velocity $-\omega$.
- **simple harmonic motion** $\equiv$ **SHM**: any motion whose time-dependence can be described by a single harmonic function.
- **simple harmonic oscillator** $\equiv$ **SHO**: any object that is undergoing simple harmonic motion.

Equations

$$x = A\cos(\omega t) \qquad F = -m\omega^2 x = -kx$$

$$T = \frac{2\pi}{\omega} \qquad E_p = \frac{1}{2}kx^2$$

$$\nu = \frac{1}{T} \qquad E_k = \frac{1}{2}k(A^2 - x^2)$$

$$\delta = \omega\, t$$

PROBLEM SUPPLEMENT

Note: Make sure your calculator is set for the correct angular units among the choices available: radians, degrees, etc. Answers are in coded order, given by bracketed letters.

Note: Problem 6 also occurs in this module's *Model Exam.*

1. Fill in the chart below for a particle in SHM, writing "min," "max," or "0" in each space. *Answer: 5*

Phase →	$5\pi/2$	3π	$7\pi/2$	4π	$9\pi/2$
x					
v					
a					
F					
E_k					
E_p					

2. An ant sits on the end of the minute hand (15.0 cm long) of a clock. Write the equation for its displacement along an axis that goes through 3:00, 9:00, and the center of the dial. *Answer: 3*

3. The displacement of a 1.0 kg mass attached to the end of a vibrating spring is given by the equation:

$$x = 0.040\,\mathrm{m}\ \cos(\pi\,\mathrm{s}^{-1}\,t)\,.$$

   a. Determine the amplitude, phase, angular frequency, frequency, and period of this motion. *Answer: 1*

   b. Determine the force, potential energy, kinetic energy and total energy when $t = 0.10\,\mathrm{s}$. *Answer: 1*

   c. Sketch the kinetic energy and potential energy, along with the displacement, as functions of time. *Answer: 1*

4. The maximum force acting on a certain 2.0 kg mass, which oscillates with SHM, is found to be 10.0 N. At that time its displacement from equilibrium is 0.10 m. Determine the angular frequency, period and frequency of its motion. *Answer: 4*

5. A particle is moving with simple harmonic motion. Its displacement at $t = 0.167\,\text{sec}$ is $0.0050\,\text{m}$ and its period is $1.00\,\text{sec}$. Determine the displacement, velocity and acceleration as functions of time. Evaluate x, v, a, and δ at $t = 0.50\,\text{s}$. *Answer: 2*

6. An object of mass $1.0\,\text{kg}$ is moving with SHM, with an amplitude of $0.010\,\text{m}$, an angular frequency of $4\pi\,\text{rad/s}$, and a maximum displacement at time zero.

   a. Write down the kinematical expression for the displacement as a function of time.

   b. Find the displacement, velocity, potential energy and kinetic energy at a time 3/8 of a period past a time when $x = 0.010\,\text{m}$.

   c. Sketch the displacement and potential energy versus time on a graph.

   d. Use the above sketch to describe how the velocity changes as position changes through a cycle.

   Answer: 6

Brief Answers:

1. a. $x = 0.040\,\text{m}\cos(\pi\,\text{s}^{-1}\,t)$

   $A = 0.040\,\text{m}$; $\omega\,t = \pi\,t$; $\omega = \pi\,\text{rad/s}$

   $\nu = \omega/(2\pi) = (1/2)\,\text{cycle/s}$; $T = 1/\nu = 2.0\,\text{s}$.

   b. $F = -m\,\omega^2\,x$; $m = 1.0\,\text{kg}$; $t = 0.10\,\text{s}$

   $F = -(1.0\,\text{kg})(\pi\,\text{s}^{-1})^2\,[0.040\,\text{m}\cos(0.10\,\pi)] = -0.38\,\text{N}$

   $E_p = (1/2)kx^2 = (1/2)m\omega^2x^2$

   $= (1/2)\,(1.0\,\text{kg})(\pi\,\text{s}^{-1})^2(0.038\,\text{m})^2 = 0.0071\,\text{J}$

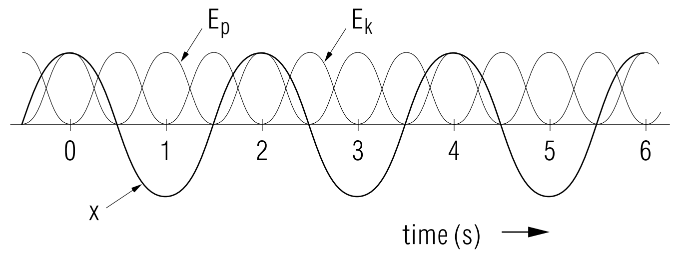
   $E_k = (1/2)(m\,\omega^2)(A^2 - x^2)$

   $= (1/2)[(1.0\,\text{kg})(\pi\,\text{s}^{-1})^2][(0.040\,\text{m})^2 - (0.038\,\text{m})^2] = 0.0008\,\text{J}$

   $E = (1/2)kA^2 = (1/2)m\omega^2A^2$

   $= (1/2)(1.0\,\text{kg})(\pi/\,\text{s})^2(0.040\,\text{m})^2 = 0.0079\,\text{J}$

c.



Help: [S-12]

2. $\omega = 2\pi\,\text{rad/s}$

   $x(t) = A\cos(\omega t)$

   $x(0.167\,\text{s}) = 0.0050\,\text{m} = A\cos(2\pi\,\text{rad/sec} \times 0.167\,\text{s})$; solve for the amplitude.

   $A = 0.0050\,\text{m}/(0.50) = 0.0100\,\text{m}$

   At all times:

   $x = 0.0100\,\text{m}\cos(2\pi\,\text{s}^{-1}\,t)$

   $v = dx/dt = -\,0.0200\,\pi\,\text{m/s}\,\sin(2\pi\,\text{s}^{-1}\,t)$

   $a = d^2x/dt^2 = -\,0.0400\,\pi^2\,\text{m/s}^2\,\cos(2\pi\,\text{s}^{-1}\,t) = -(2\pi/\,\text{sec})^2\,x$

   At t = 0.50 s:

   $x = 0.0100\,\text{m}\cos(\pi) = -0.0100\,\text{m}$

   $v = 0.000\,\text{m/s}$

   $a = 0.395\,\text{m/s}^2$

   $\delta = \omega\,t = 1.8 \times 10^2$ degrees

3. $x = A\cos\,\omega t$

   $\omega = 2\pi/T = 2\pi/60\,\text{min} = 0.10/\,\text{min}$ *Help: [S-13]*

   $x = 15.0\,\text{cm}\cos(0.10\,\text{min}^{-1}\,t)$

4. $F = -\,k\,x$

   $k = -F/x = -(-10.0\,\text{N})/0.10\,\text{m} = 1.0 \times 10^2\,\text{N/m}$

   $\omega = (k/m)^{1/2} = (100\,\text{N}\,\text{m}^{-1}/2.0\,\text{kg})^{1/2} = 7.1\,\text{rad/s}$

$T = 2\pi/\omega = 2\pi/(7.1/\,\mathrm{s}) = 0.88\,\mathrm{s}$

$\nu = 1/T = 1/0.88\,\mathrm{s} = 1.1\,\mathrm{cycles/s} = 1.1\,\mathrm{Hz}.$

5.

Phase →	$5\pi/2$	3π	$7\pi/2$	4π	$9\pi/2$
x	0	min	0	max	0
v	min	0	max	0	min
a	0	max	0	min	0
F	0	max	0	min	0
E_k	max	0=min	max	0=min	max
E_p	0=min	max	0=min	max	0=min

6. a. $x = 0.010\,\mathrm{m}\cos(4\pi\,\mathrm{rad/s}t)$

b. $T = (1/2)\,\mathrm{s}.$

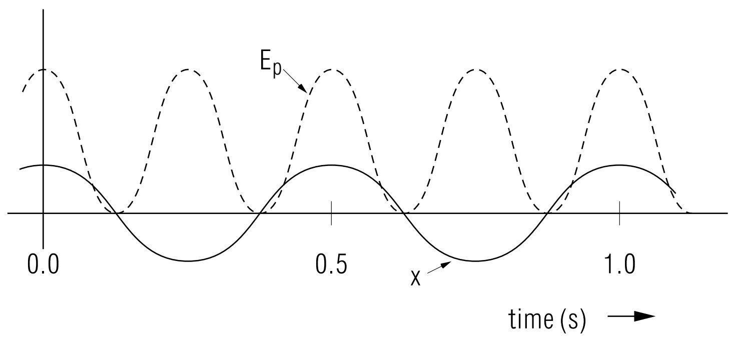
at $t = (3/16)\,\mathrm{s}$:

$x = -0.0071\,\mathrm{m}$ (see graph below)

$v = -0.089\,\mathrm{m/s}$ (see slope of graph below)

$E_p = 0.0040\,\mathrm{J}$ (see graph below)

$E_k = 0.0040\,\mathrm{J}$



d. See text.

SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from TX-2b)*

$x = A\cos(\omega\,t)$ and $\cos(0) = 1$

S-2 *(from TX-2c)*

$\omega = 4\pi\,\mathrm{rad/s}$; $A = 5.0\,\mathrm{cm}$
$x = A\cos(\omega\,t)$
$\quad = 5.0\,\mathrm{cm}\;\cos(4\pi\,\mathrm{rad\,s^{-1}\,t})$
at $t = 1.0\,\mathrm{s}$: $x = 5.0\,\mathrm{cm}\cos(4\pi) = 5.0\,\mathrm{cm}$
Note: $\cos n2\pi = 1$ when n is an integer.

S-3 *(from TX-2d)*

$\omega = 4\pi\,\mathrm{rad/s}$
$T = 2\pi/\omega = 2\pi\,\mathrm{rad}/(4\pi\,\mathrm{rad\,s^{-1}}) = 0.5\,\mathrm{s}$
$\nu = 1/T = 1/0.50\,\mathrm{s} = 2.0\,\mathrm{cycles/s} = 2.0\,\mathrm{Hz}$

S-4 *(from TX-3c and [S-11])*

"As the oscillator passes the origin, the energy is all kinetic; the potential energy is zero and the kinetic energy is at its maximum. As the displacement increases positively, the potential energy increases so the kinetic energy decreases in order to keep the total energy constant. When the kinetic energy reaches zero, the displacement is at its maximum value and the energy is all potential. As the displacement starts decreasing, ... "

S-5 *(from TX-3a)*

"As time advances, the point moves along the line, from upper left to lower right and back again. As it passes the origin, where the displacement and acceleration are zero, the point is moving at maximum speed. As it approaches an end, where both the position and acceleration are at their maximally positive or negative values (extrema), the point slows down, stops, and reverses its direction."

S-6 *(from TX-2e)*

Note that the cosine function is at its maximum at $t = 0$ and thereafter decreases, crossing the axis one-fourth of a period later and reaching its maximally negative value one-fourth of a period after that. Use these characteristics to sketch in the curve in those regions. Continue drawing the curve forward in time, making sure the curve crosses the axis each half period, as a cosine curve always does. For the other curves, use their equations and the same procedure.
If you don't know what sine and cosine curves look like, see your math textbooks from past math courses in college or high school.

S-7 *(from TX-3a)*

This is just simple algebra!

S-8 *(from TX-2f)*

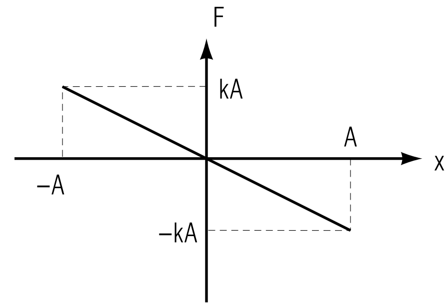
The point is claimed to follow a circle with radius $r = A$ and a time-changing angle: $\theta(t) = -(\omega t)$. This means that Eqs. (2) and (5) can be written this way in polar coordinates:
$x(t) = r\cos\theta(t); \quad y(t) = r\sin\theta(t)$.
These are the equations for a point on a circle, so the claim is proved. Now note that, as time increases, θ increases negatively. Since positive polar angles are measured counterclockwise from the x-axis, and negative angles clockwise, the point moves clockwise as time increases.

S-9 *(from TX-2f)*

First, see [S-8]. The angular velocity of the plotted point is:
$d\theta/dt = d/dt(-\omega t) = -\omega$.
This shows that the magnitude of the angular velocity of the plotted point is just the ω of the oscillation, but the minus sign means that the direction with increasing time is clockwise. That is, the plotted angle increases negatively with time.

S-10 *(from TX-3a)*

The force constant, k, is the negative of the slope of this line:



S-11 *(from TX-3c)*

See [S-4].

S-12 *(from PS-2c)*

If you are having trouble with this problem, do Problem 4 first and then redo the "try-it" in Sect. 2d.

S-13 *(from PS-3)*

The period of the minute hand is 60 minutes because it takes one hour for it to go through a complete cycle (once around the clock face).

S-14 *(from TX-2b)*

As its argument increases from zero, a sine function increases positively from zero. Thus as t increases from zero, x increases from zero. Increasing values of x correspond to the direction *upward*.

S-15 *(from TX-2c)*

The units of ν are oscillations per unit time while the units of ω are angular interval per unit time. As commonly used ν is in cycles/sec while ω is in radians/sec. This is because the "2 π" you use to convert one to the other is really "2 π radians (the angular interval around a complete circle) per complete oscillation."

MODEL EXAM

1. See Output Skills (Knowledge) on this unit's *ID Sheet*.

2. An object of mass 1.0 kg is moving with SHM, with an amplitude of 0.010 m, an angular frequency of 4π rad/s, and a maximum displacement at time zero.

   a. Write down the kinematical expression for the displacement as a function of time.

   b. Find the displacement, velocity, potential energy and kinetic energy at a time 3/8 of a period past a time when $x = 0.010$ m.

   c. Sketch the displacement and potential energy versus time on a graph.

   d. Use the above sketch to describe how the velocity changes as position changes through a cycle.

Brief Answers:

1. See text.

2. See this module's *Problem Supplement*, problem 6.