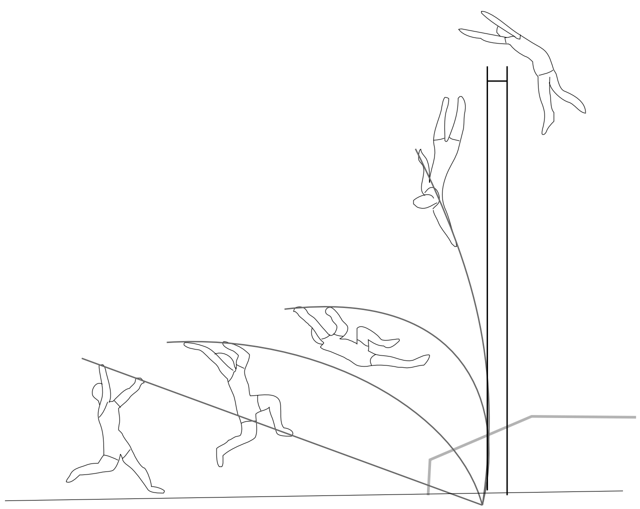
PHYSNET

MISN-0-21

POTENTIAL ENERGY, CONSERVATION OF ENERGY



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

POTENTIAL ENERGY, CONSERVATION OF ENERGY

by

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Title: **Potential Energy, Conservation of Energy**

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Input Skills:

1. Vocabulary: friction (MISN-0-16), line integral (MISN-0-20).
2. Calculate the work done by a force along a prescribed path.
3. State Newton's second and third laws (MISN-0-14).
4. Solve a system of n simultaneous equations in n unknowns (MISN-0-1) and calculate the dot (scalar) product of any two vectors (MISN-0-2).

Output Skills (Knowledge):

K1. Define the potential energy function associated with a conservative force. Explain what a "conservative" force conserves, using examples of both conservative and non-conservative forces.

Output Skills (Problem Solving):

S1. Given any one dimensional conservative force, determine the potential energy function using standard reference points.

S2. Solve problems involving both conservative and non-conservative forces using the general form of the law of conservation of energy.

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1. Introduction

In this module we are primarily concerned with the concept of conservation of energy. We first introduce the concept of potential energies to replace those line integrals in the work definition that produce path-independent work. We then state the Law of Conservation of Energy, which is perhaps the most useful unifying concept of science.

Understanding Conservation of Energy will give you insight into many ordinary experiences from everyday life (such as why some people get fat!). You will also be better prepared to analyze the "energy crisis" and some of the proposed cures. (Don't waste time with schemes that promise more energy out than you put in!) In addition, you will develop an extremely powerful tool for solving a large class of physics problems with a minimum of mathematical manipulation. In order to further restrict the mathematics needed, we shall only deal with one-dimensional motion in this module.

2. Potential Energy

2a. Overview. When work is performed on a system,[1] and the energy thus transferred into the system is stored internally in such a way that it can later be converted back to mechanical energy by reversing the storage process, we call the stored energy "potential energy." That is because it has the *potential* to be recovered by reversing the storage process. Good examples are the energy stored through raising water against the force of gravity and the energy stored through compressing a spring in a mechanical clock. In the case of the elevated water the stored energy can be reclaimed to run electrical generators while in the case of the clock spring the stored energy is used to move the hands.

2b. Frictional Forces. Work done against *the force of friction* does not cause a gain in mechanical *potential* energy because the process is not

[1]See "Work, Power, Kinetic Energy" (MISN-0-20).

mechanically reversible. For example, running a car at constant speed in reverse gear does not enable you to regain the mechanical energy you previously expended running the car at the same speed in forward gear. In fact, running the car in reverse gear requires the same *expenditure* of energy as running it in forward gear. While running the car at constant speed, in either gear, *all* of the energy expended goes into overcoming friction. *Help: [S-2]*

2c. Conditions for Potential Energy. There are several *equivalent* ways of describing the situations in which potential energy can be gained or lost. Two of them are: (i) the work done is reversible; and (ii) the work integral is independent of path.

2d. A Conservative Force. When the above conditions are fulfilled, the storage force involved in the energy storage is universally said by professionals to be a "conservative" force. By this they mean that the energy involved is conserved, with no losses, as *mechanical* energy. The energy-storage force is *not* the force you apply as you do work on the system; rather, it is an opposing force that is internal to the system, a loss-less mechanical storage force. The main two examples we will deal with here are the force of gravity and the restoring force of springs.

Whether the force is "conservative" or not, *total* energy is *always* conserved.

2e. Potential Energy Difference. The work done *by* a lossless mechanical storage force whose point of application moves from A to B is:

$$W_{A\to B} = \int_A^B \vec{F}_c \cdot d\vec{s}.$$

where the subscript "c" explicitly shows that the force is a "conservative" one. The negative of this work is the work done *on* the force and hence is the change in stored energy. We call this change the "potential energy difference, ΔE_p, at point B relative to point A":[2]

$$\Delta E_p \equiv -\int_A^B \vec{F}_c \cdot d\vec{s} = -W_{c,A\to B} \tag{1}$$

Note the significance of the signs of the terms: because the line integral does not depend on path, $W_{A\to B} = -W_{B\to A}$: i.e., the work done against $\vec{F}$ in moving from A to B can be retrieved by moving back from B to A.

[2]Some authors use U, V or PE. Our notation, E_p, emphasizes that this is just a particular form of energy.

Thus the net path-independent work done around any *closed* path is *zero*. Actually, this should be obvious since one possible path from a point A to the same point A is no movement at all and that certainly involves zero work.

2f. Path-Dependent Work. When a non-conservative force, $\vec{F}_{nc}$, is involved, the value of the line integral,

$$\int_A^B \vec{F}_{nc} \cdot d\vec{s},$$

depends on the actual path of integration. *Help: [S-1]* The integration path in such cases must follow the actual physical path taken by the point of application of the force.

3. The Potential Energy Function

3a. Standard Reference Points. We often dispense with the cumbersome $A \to B$ terminology and choose a "standard reference point," a point which often turns out to be the lower limit of the integral in practical problems. We then integrate from this point as the lower limit to an arbitrary space point (x,y,z) as the upper limit of the integral. The resulting function of (x,y,z) is called the "potential energy function" and, for an object at (x,y,z), $E_p(x,y,z)$ is called the object's "potential energy."

If you have a potential energy function and you wish to know the difference in potential energy between a space point and the reference point, merely evaluate the function at the point in question. If you wish to know the difference in potential energy between the point in question and a point that is not the reference point, merely evaluate the function at both points and take the difference. If you see a potential energy function and you are uncertain as to the reference point that was used to generate it, just set the potential energy function equal to zero and solve for (x,y,z). That will be the reference point that was used.

3b. Gravitational Potential Energy. Near the surface of the earth, the gravitational force on mass m (i.e., its weight) is essentially constant and is given by $m\vec{g}$. Choosing a coordinate system with unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$ along east, north and up, respectively, and choosing the coordinate origin (wherever we might have put it) as the reference point, the gravitational potential energy at the point z_0 is:

$$E_{p,\mathrm{grav}}(z_0) = -\int_0^{z_0} m(-g)\,dz = mg\int_0^{z_0} dz = mgz_0\,.$$

where z_0 is height above the coordinate origin (which we have taken as the zero of potential energy for this case). We usually prefer to talk about the potential energy at some point z so we make a change of notation in the equation above to get the usually-used form:

$$E_{p,\mathrm{grav}}(z) = -\int_0^{z} m(-g)\,dz' = mg\int_0^{z} dz' = mgz\,. \qquad (2)$$

where z is now height above the coordinate origin, our chosen zero of potential energy. The variable z' is called the "dummy variable of integration." *Help: [S-30]*

Suppose an object of mass m is located at the point (x_1,y_1,z_1) and is then moved to the point (x_2,y_2,z_1+h). Using Eq. (2) we find that its increase in gravitational potential energy is just mgh. Note also that this is equal to the work done by an outside force $\vec{F} = mg\hat{z}$ that raises the mass m through height h against the gravitational force that is pulling the mass and the earth toward each other. So mgh is called the gravitational potential energy of the mass m at height h above some coordinate origin, the reference level we have chosen.

Thus a typical concrete block with a weight of 175 N will have a potential energy of 175 N m relative to your toe if you are holding it at a height of 1.00 meter above your toe.

Note that the gravitational potential energy in Eq. (2) is *not* a property of the object of mass m: instead, it is a property of the object-plus-earth system and that system's gravitational interaction. The gravitational potential energy of that system increases when the distance between the object and the earth is increased, and it decreases when that distance is decreased.

3c. Other Illustrative Forces.

1. Consider a conservative force of the form $F_c = -kx$, a spring-type force, acting on some particle. Using Eq. (1):

$$E_p = -\int_0^{x} -kx'\,dx' = kx^2/2. \qquad (3)$$

   Just looking at the result, it is clear that the reference point is $x = 0$ since that is the point where $E_p = 0$.

2. Consider $F_c = k/r^2$. Using Eq. (1):

$$E_p = -\int_\infty^{r} k/r'^2\,dr' = k/r. \qquad (4)$$

The reference point, the point where $E_p = 0$, is at infinity, so E_p represents the work that is done by F_c on a particle as it moves from any point at radius infinity to any point at radius r.

4. Conservation of Energy

Many of the great turning points in the history of physics have been characterized by the discovery of situations that appeared at first to violate Eq. (1). But, in every case, it has been found that some form of E_k or E_p had been overlooked and total energy was actually conserved. We can write Conservation of Energy this way:

$$\boxed{\begin{array}{c}\text{The total energy in a system at time } t\\ +\\ \text{the energy entering the system during time } \Delta t\end{array}}$$
$$= \qquad (5)$$
$$\boxed{\begin{array}{c}\text{the total energy in the system at time } t+\Delta t\\ +\\ \text{the energy leaving the system during time } \Delta t.\end{array}}$$

Note the close analogy with your financial affairs:

$$\boxed{\begin{array}{c}\text{Your total money at the beginning of some time interval}\\ +\\ \text{money you acquired during the time interval}\end{array}}$$
$$=$$
$$\boxed{\begin{array}{c}\text{money you have at the end of the time interval}\\ +\\ \text{money you spent during the time interval.}\end{array}}$$

Just as "the money you have" at a given time may be stored in various places and in various forms, so "the energy in a system" may be stored in many forms. *Help: [S-23]* Fortunately, we need only consider those forms that change for systems we study during the time interval being considered. We will only study two forms here: kinetic energy and mechanical potential energy.

▷ Rephrase each of the four terms in Eq. (5) in terms of the change in a system's kinetic energy, ΔE_k, the change in its potential energy, ΔE_p, and the work done *on* the system by non-conservative forces, $W_{nc,on}$. *Help: [S-16]*

▷ Suppose we have a 60.0 kg box that we wish to unload from a truck. Suppose this will require lowering the box through a height of 1.0 m. Use kinematics to find the speed of the box just before hitting the ground if the box is simply pushed off the end of the truck. *Help: [S-14]*

▷ In the above exercise assume that the velocity of impact is unacceptable, so we must find a gentler way of unloading the box. It is too heavy to lift by hand, but the truck is equipped with a ramp that is 3.0 m long. Use Eq. (5) and proper problem solving techniques to find the average force up the incline, F, that must be applied to the box to let it arrive at the bottom of the incline with negligible speed. *Help: [S-15]*

5. Examples

5a. Energy in the Morning.

$$\boxed{\begin{array}{c}\text{Your total energy in the morning}\\ +\\ \text{energy taken in during the day (from food, etc.)}\end{array}}$$
$$=$$
$$\boxed{\begin{array}{c}\text{your total energy at the end of the day.}\\ +\\ \text{energy leaving during the day (work, heat, etc.)}\end{array}}$$

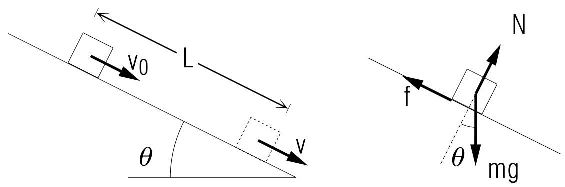
Note that significant increases in internal energy are stored as fat!

5b. Concrete Block Drops on Toe. Consider what would happen if you let go of the concrete block mentioned near the end of Sect. 3b. It started with 175 J of gravitational E_p relative to your toe. The only significant force on the block while falling is mg, which is conservative, so when the block first touches your toe it will have $E_k = 175$ J. A short time later the block is at rest at $h = 0$ so it has $E_k = 0$ and $E_p = 0$. At first glance it appears that 175 J of energy was "lost." Actually, this energy went into breaking chemical bonds (change in chemical potential energy) and into random kinetic energy of molecules (increase in temperature).

5c. Box Slides Down Incline. A box slides 5.0 m down a 36.87° incline in a region where g is 9.8 m/s^2. The velocities of the box at the beginning and end of this interval are 4.0 m/s and 6.0 m/s, respectively, down the incline.

▷ Find the coefficient of friction, assuming it is independent of velocity:

$L = 5.0\,\mathrm{m}$
$\theta = 36.87°$
$v_0 = 4.0\,\mathrm{m/s}$
$v = 6.0\,\mathrm{m/s}$
$g = 9.8\,\mathrm{m/s^2}$
$\mu = ?$



From Eq. (5):

$$(mgh + \frac{1}{2}mv_0^2) + 0 = \frac{1}{2}mv^2 + fL,$$

where f is the frictional force and h is $L\sin\theta$. From Newton's second law:

$$N = mg\cos\theta.$$

Then:

$$\begin{aligned}\mu = \frac{f}{N} &= \frac{mg\sin\theta - m(v^2 - v_0^2)/(2L)}{mg\cos\theta}\\ &= \tan\theta - \frac{(v^2 - v_0^2)}{2Lg\cos\theta}\\ &= \frac{3}{4} - \frac{(36\,\mathrm{m^2/\,s^2} - 16\,\mathrm{m^2/\,s^2})}{2(5\,\mathrm{m})(9.8\,\mathrm{m/s^2})(0.80)} = 0.49\,.\end{aligned}$$

5d. Other Examples. Potential energy curves are useful in analyzing motion and can be used to display complete information about the dynamical motion of a one-dimensional conservative system.[3] The application of conservation of energy to fluids is the basis of fluid dynamics.[4] Conservation of energy is used to determine the motion of oscillating systems.[5] Energy concepts are applied to the analysis of the rotation of rigid bodies.[6]

Acknowledgments

I would like to thank Richard Hagan, Angelite Arnold, Richard Jones, and Tom Hudson for their helpful reviews of this module. Preparation of

[3]See "Potential Energy and Motion; Potential Curve, Turning Points" (MISN-0-22).
[4]See "Fluids, Static and Dynamic" (MISN-0-48).
[5]See "Simple Harmonic Motion" (MISN-0-25).
[6]See "Translational and Rotational Motion of a Rigid Body" (MISN-0-43).

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Some Symbols Used

A, an arbitrary fixed point

B, an arbitrary fixed point

$d\vec{s}$, a differential displacement

E_k, kinetic energy

ΔE_k, change in kinetic energy

E_p, potential energy

$\mathrm{E}_{p,grav}$, gravitational potential energy

ΔE_p, change in potential energy

$\vec{F}_c$, conservative force

$\vec{F}_{nc}$, non-conservative force

$\vec{F}$, force

$\vec{f}$, frictional force

g, acceleration of gravity

h, height

m, mass

$\vec{r}$, radius

$\hat{r}$, unit vector in the radial direction (away from origin)

$W_{A\to B}$, work done by a force moving from A to B

$W_{nc,on}$, the work done on a system by non-conservative forces

$\hat{x}$, a unit vector in the x-direction

PROBLEM SUPPLEMENT

Use: $g = 9.8\,\mathrm{m/s^2}$

Use Conservation of Energy in each problem marked with an asterisk (*).

If no Reference Point is given, use the standard one for that type of problem.

-Problems 20-22 are also in this module's *Model Exam.*

1. In the example of Sect. 5b in the text, we found that 175 J of energy was converted to heat and breaking of chemical bonds, primarily in your toe. If it requires an average of 10^{-18} J to break one molecular bond and 4/7 of the "lost" energy went into breaking molecular bonds, how many bonds were broken?* *Answer: 3*

2. The useful output power from a certain water pump is 80.0% of the electrical power supplied to it. This pump is used to raise 4.0 kg/s of water from a well. If the water leaves the system at a speed of 8.0 m/s at a height of 12.0 m above the water level in the well, find the electrical power being supplied to the pump.* *Answer: 7 Help: [S-24]*

3. The high-dive at an Olympic swimming pool is 10.0 m (32.8 ft) above water level. Neglecting air resistance, with what speed will a diver hit the water if he or she starts with a speed of 5.0 m/s at an angle of 36.87° above the horizontal?* *Answer: 1 Help: [S-25]*

4. A girl of mass 49.0 kg is on a swing which has a mass of 1.0 kg. Suppose you pull her back until her center of mass is 2.0 m above the ground. You let her go and she swings out and returns to the same point. Are all forces acting on the system conservative? (Caution: this question deserves a carefully qualified answer.) *Answer: 2*

5. Assume that all forces in the previous problem are conservative, and that the center of mass of the girl-plus-swing system is 0.75 m off the ground at its lowest point. Find the girl's maximum speed.* *Answer: 8*

6. Suppose you push the girl in problem 5 and add 490 J of energy to the system. Find her new maximum height above the ground.* *Answer: 4*

7. A certain toboggan with its riders has a mass of 305 kg and goes down a hill which is 25 m high. The toboggan starts from rest and attains a final speed at the bottom of the hill of 12 m/s. Find the energy lost to friction.* *Answer: 10*

8. One bit of advice that bowlers often hear is "let the weight of the ball do the work. Don't force it." For each of the following two cases, find the percentage of the ball's final energy which is supplied by the bowler:* *Help: [S-26]*

   a. Good bowler, average = 200, backswing height = 1.10 m, release speed = 6.76 m/s. *Answer: 5*

   b. Average bowler, average = 120, backswing height = 0.70 m, release speed = 7.70 m/s. *Answer: 5* (Data from Murose, "Biomechanics of Bowling," Nelson, ed., *Biomechanics IV*, 1974.

9. A rubber ball is dropped from a height of 1.5 m onto a concrete floor and rebounds to a height of 1.2 m. What fraction of its original energy was lost to friction?* *Answer: 11*

10. Given a conservative force defined by $\vec{F} = -kx^2\hat{x}$ find the corresponding potential energy function. State the reference point. *Answer: 6*

11. Given a conservative force, $\vec{F} = k/r^3\hat{r}$, find the associated potential energy function. State the reference point. *Answer: 9*

12. Given the conservative force $\vec{F} = xy_0\hat{x}$ find the associated potential energy function. State the reference point. *Answer: 13*

13. Given the conservative force, $\vec{F} = A\cos x\,\hat{x}$, find the potential energy function. State the reference point. *Answer: 14*

14. A man at the scene of an accident claims that he was traveling on a level road at 25 m/s (56 mph) when he saw the car pull out in front of him. He says he skidded for about 30 m and hit the other car with a speed of about 5 m/s. If he is telling the truth, what was the coefficient of friction, assuming it to be independent of speed? *Answer: 3 Help: [S-12]*

15. A car with a mass of 1500 kg accelerates from rest to a speed of 21 m/s in a time of 8.0 s. Neglecting losses due to friction and assuming a level road, what horsepower is required? (hp = 746 W) *Answer: 15*

16. Re-do the previous problem for the car on a hilly road if its increase in height is 8 m.* *Answer: 17*

17. A 40.0 kg mass is traveling horizontally with velocity $2.0\,\hat{x}$ m/s as it crosses the coordinate origin and encounters the force $\vec{F} = -kx\hat{x}$ due to a spring. If $k = 250$ N/m, at what value of x will the mass stop?* *Answer: 3 Help: [S-18]*

18. A system consists of particles A and B with masses 11 kg and 21 kg respectively. At time zero: A has a velocity of (5.0 m/s, east) at height $h = 0.0$. B has a velocity of (4.0 m/s at 23° north of east) and $h = 2.0$ m. One hour later: A is at rest at $h = 3.0$ m. B has a velocity of (8.0 m/s, north) at $h = 0.0$. Find the net work done on the system during the hour.* *Answer: 16 Help: [S-29]*

19. How many times could a 72 kg person climb the Empire State Building while using up 2.0×10^3 food calories of energy, if her or his body were 100% efficient in turning food energy into useful mechanical energy? The Empire State Building is 381 m high, and a food calorie = 4186 J. *Answer: 19* (It should be clear from the answer to this problem that very little of the body's energy is converted into useful mechanical energy. Most of it goes into heat.)

20. A nonconstant force, F, directed vertically upward (the force varies in magnitude as some unknown function of altitude) is exerted on an object of mass m. Under the action of this force, the object moves straight up from ground level to a distance h, above the ground. It starts at rest and when it gets to height h it is moving with speed v upward.

    a. Draw the free-body diagram of forces that act on this mass during the course of its motion upward. Label the unknown force vector with a symbol but point it in the proper direction. *Answer: 35*

    b. Write down the Work-Force integral for this system, expressing the work of each of the forces separately. *Answer: 22*

    c. Evaluate the integrals that can be evaluated. *Answer: 28*

    d. Draw the appropriate picture(s) for solving this problem using conservation of energy. *Answer: 20*

    e. Write the conservation of energy relation for this object. *Answer: 32*

    f. How much work was done by the force F? *Answer: 34*

21. A nail partially driven into a board, is driven in further by dropping a mass onto it. The mass, m, is released a distance y_0 above the head of the nail and drives the nail an additional distance s into the board. Neglect the mass of the nail.

    a. Defining the zero of potential at the top of the nail, find the potential energy of the mass m at the instant m is released. Assume the nail's mass is negligible. *Answer: 23*

    b. What is the kinetic energy at that instant? *Answer: 33*

    c. Find the potential energy after the mass has driven the nail the additional distance s into the board. *Answer: 29*

    d. What is the kinetic energy at that instant? *Answer: 21*

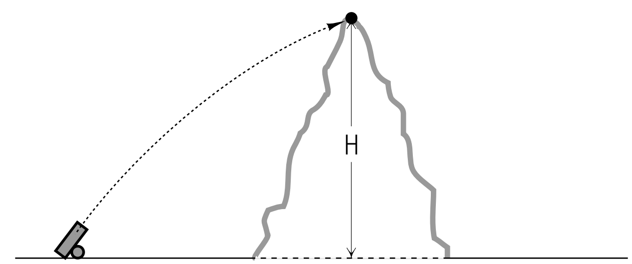
    e. Write the conservation of energy relation for this problem. *Answer: 25*

    f. How much work is done on the mass m by the nail-plus-board-plus-earth mechanically resistive system? *Answer: 30*

    g. How much work is done on the nail-plus-board-plus-earth mechanically resistive system by the mass-plus-earth gravitational system? *Answer: 31*

    h. Assuming that the nail exerts a constant force on the mass during the interval that the nail is slowing the mass to a stop, determine the magnitude of this force, F_N.* *Answer: 24*

22.



An artillery shell fired from a gun at ground level leaves the muzzle of the gun with a speed of 5.0×10^1 m/s. The shell strikes a direct hit at a site on the top of a hill which is at an altitude of 45 m above ground level.

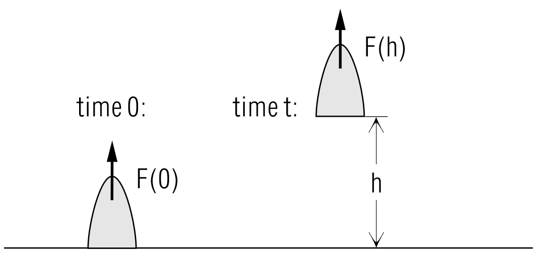
    a. Assuming no friction, use energy conservation to find the speed with which the shell hits at the site. *Answer: 27*

    b. If the shell strikes the ground at the site with a speed of 38 m/s, what percentage of its original energy was lost (to friction with the air) by the shell in its flight? *Answer: 26*

Brief Answers:

1. 14.9 m/s
2. Possibly yes, if there is no friction. But in the more likely real case, there will be friction, which is a non-conservative force. Thus if she does indeed return to the same point, there must have been a non-conservative force acting on the system, e.g., the girl "pumping."
3. 10^{20}. Ouch!
4. 3.0 m
5. a) 53%; b) 77%
6. $kx^3/3$; $x = 0$
7. 7.5×10^2 W
8. 5.0 m/s
9. $k/(2r^2)$; $x = \infty$
10. 53 kJ
11. 1/5
12. 1.0
13. $-x^2 y_0/2$; $x = 0$
14. $-A \sin x$; $x = 0$
15. 55 hp
16. 2.8×10^2 J
17. 75 hp
18. 0.80 m
19. 31 times

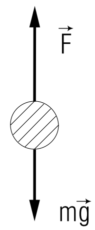
20.



21. Zero
22. $\int_{\text{ground}}^{h} \vec{F} \cdot d\vec{r} + \int_{\text{ground}}^{h} (-mg)ds = \frac{1}{2}mv^2$
23. mgy_0
24. $mg\left(1 + \frac{y_0}{s}\right)$
25. $mgy_0 = -mgs+$ energy dissipated in heat and ruptured bonds
26. $\approx 7\%$ (6% or 7%)
27. 4.0×10^1 m/s
28. Second integral is $-mgh$
29. $-mgs$
30. $-mg(y_0 + s)$
31. $mg(y_0 + s)$
32. $\int_{\text{ground}}^{h} \vec{F} \cdot d\vec{r} = \frac{1}{2}mv^2 + mgh.$

    Note that $E_k(\text{ground}) = 0$ and $E_p(\text{ground}) = 0$.
33. Zero
34. $\frac{1}{2}mv^2 + mgh$

35. See diagram:



SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from TX-2f)*

Write the general definition for the work W done by force F in moving from A to B.

* STOP * & WRITE THE ANSWER BEFORE PROCEEDING!

Help: [S-7]

S-2 *(from TX-2b)*

Let $W_{nc} \equiv \int_A^B \vec{F}_{nc} \cdot d\vec{s} \equiv$ the work done by a non-conservative force. You will then have different values for W_{nc} depending on the actual path taken between points A and B. For example, consider moving a 50 kg box from point A to point B on a level floor, if B is 4 m north and 3 m east of A. Assume the coefficient of friction between the box and the floor is $\mu = 0.40$. If you apply a horizontal force $\vec{F}$, find the work which you do in moving the box from A to B.

This problem has as many different answers as there are paths between A and B. The shortest path is the straight line from A and B of length 5 m. The only horizontal forces are F and f, i.e., the applied force and friction. If we assume the initial and final velocities are zero, the average acceleration must be zero. It follows from Newton's second law that the average resultant force must be zero, so $\vec{F} = -\vec{f} = \mu\,\mathrm{N}\,\hat{s}$. The only vertical forces are N up and mg down, so again from Newton's second law, $N = mg$. Thus $\vec{F} = \mu mg\hat{s}$. Then we have

$$W = \int_A^B \mu mg\,\hat{s} \cdot d\vec{s} = \mu mg \int_A^B ds = \mu mgS,$$

where S is the total distance traveled between A and B. Clearly S could be any number ≥ 5 m depending on the path; e.g., for the shortest path, $W = (0.4)(50)(9.8)5 = 980$ J. But if you traveled 4 m north and then 3 m east, $S = 7$ m, and $W = 1372$ J.

S-5 *(from Help: [S-15])*

$f = F = 200$ N when $F = \mu mg\cos\theta$. Here θ is the angle the normal to the incline makes with the vertical. It is also the angle the incline makes with the horizontal. Now $\cos\theta = (L^2 - h^2/L)^{1/2}$. So $\mu = FL/\left(mg(L^2h^2)^{1/2}\right) = 1/(8)^{1/2} = 0.35$.

$\boxed{\text{S-7}}$ *(from Help: [S-1])*

$$W \equiv \int_A^B \vec{F} \cdot d\vec{s}$$

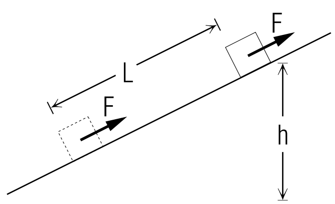
If you did not put down exactly this result, review MISN-0-20.

$\boxed{\text{S-14}}$ *(from TX-4)*

$m = 60.0\,\text{kg}$, $v_0 = 0$, $g = 9.8\,\text{m/s}^2$, $h = 1.0\,\text{m}$, $v = ?$
From kinematics (See "Kinematics in One Dimension," MISN-0-7): $v^2 - v_0^2 = 2as$, so in the y-direction, for this problem, $v^2 = 2gh$. So $v = (2gh)^{1/2} = 4.4\,\text{m/s}$.

$\boxed{\text{S-15}}$ *(from TX-4)*

$$\begin{aligned} h &= 1.0\,\text{m};\ m = 60.0\,\text{kg} \\ L &= 3.0\,\text{m}; \\ v &= 0 \\ F &= ? \\ \Delta E_k &= 0.\ \ \Delta E_p = 0 - mgh. \end{aligned}$$



$W_{nc} = -FL$, where the negative sign comes from the fact that the force F is in the opposite direction to the displacement. Alternatively, one could say that the negative sign comes from the fact that the work done by the non-conservative force is work done on the environment (including the block) by the block's surface.
Thus from Text Eq. (3) we have: $-FL = 0 - mgh$, from which $F = mgh/L = 2.0 \times 10^2\,\text{N}$.
Most of this force will probably be supplied by friction. Indeed, if the coefficient of friction is greater than 0.35 you will need to push the box if you want it to go down the incline! *Help: [S-5]*
Note that the work done by F is $-600\,\text{J}$ and the work done by gravity is $+600\,\text{J}$, so the total work done on the box is zero, which equals ΔE_k.

$\boxed{\text{S-16}}$ *(from TX-4)*

Here are various correct equations:

$$E_k(t) + \Delta E_k = E_k(t + \Delta t),$$

$$E_p(t) + \Delta E_p = E_p(t + \Delta t),$$

$$E_k(t) + E_p(t) + \Delta E_k + \Delta E_p = E_k(t + \Delta t) + E_p(t + \Delta t),$$

$$\Delta E_k + \Delta E_p = W_{nc,on}.$$

Here ΔE_k is the *net* change in the system's kinetic energy over the time interval Δt, ΔE_p is the *net* change in the system's potential energy over that same time interval, and $W_{nc,on}$ is the work done *on* the system by non-conservative forces during that same interval.

$\boxed{\text{S-22}}$ *(from TX-3b)*

Consider the particles A and B. Here are some of their properties (the F's are different forces):

Property	Time	A	B
kinetic energy	0	7.0 J	0.0 J
kinetic energy	t	2.0 J	4.0 J
potential energy due to F_1	0	3.0 J	0.0 J
potential energy due to F_1	t	1.0 J	11.0 J
potential energy due to F_2	0	0.0 J	5.0 J
potential energy due to F_2	t	0.0 J	8.0 J

Problem: find the work W done on the system.
We find: $(7 + 0 + 3 + 0 + 0 + 5)\,\text{J} + W = (2 + 4 + 1 + 11 + 0 + 8)\,\text{J}$.
$W = 11\,\text{J}$.

S-23 *(from TX-4)*

Let us further develop the analogy following Text Eq. (5). Kinetic energy is like cash. Potential energy is like a checking account — one for each conservative force acting in the problem. Moving money from one checking account to another or to cash does not change your total assets. But you must not include money taken from a checking account as "money acquired" during time Δt. Similarly, the total energy in a free falling ball is constant; energy is being taken out of the gravitational potential energy checking account and turned into kinetic energy (cash) but the total assets are constant. Looked at from this point of view, the work done by gravity while the ball falls is not energy entering the system (not money acquired) since it was already there as potential energy (i.e., in the checking account) at the beginning of the time interval.

S-24 *(from PS-prob. 2)*

Write conservation of energy between points A and B: at point A, water is at rest at the bottom of the well; at point B, it is at height h and moving with speed v. Recall that power is energy (here, work) per unit time.

S-25 *(from PS-prob. 3)*

At $t = 0$, the diver has potential and kinetic energy. Note that the result is independent of θ.

S-26 *(from PS-prob. 8)*

The unknown is W_m/E_{kf} where W_m is the work done by the man and E_{kf} is the final kinetic energy.

S-27 *(from PS-prob. 14)*

Recall that the frictional force is the coefficient of friction times the normal force.

S-28 *(from PS-prob. 17)*

Use conservation of energy to show that $x = v_0(m/k)^{1/2}$.

S-29 *(from PS-prob. 18)*

See [S-22] for a similar problem.

S-30 *(from TX-3b)*

We changed to primed symbols for the variables of integration because we wanted to use unprimed symbols for the upper limit of integration and hence for the function on the left side of the equation (our real goal).

The term "dummy variable of integration" is standard usage. It indicates that the symbol used for the variable of integration is immaterial in a definite integral because it does not appear in the final answer. Thus

$$\int_0^1 x\,dx \qquad \text{and} \qquad \int_0^1 y\,dy$$

give exactly the same answer.

MODEL EXAM

1. See Output Skill K1 in this module's *ID Sheet*.

2. A nonconstant force, F, directed vertically upward (the force varies in magnitude as some unknown function of altitude) is exerted on an object of mass m. Under the action of this force, the object moves straight up from ground level to a distance h, above the ground. It starts at rest and when it gets to height h it is moving with speed v upward.

   a. Draw the free-body diagram of forces that act on this mass during the course of its motion upward.

   b. Write down the Work-Energy relation (in its original form without potential energies) for this system, expressing the work of each of the forces separately.

   c. Evaluate the integrals that can be evaluated.

   d. Draw the appropriate picture(s) for solving this problem using conservation of energy.

   e. Write the conservation of energy relation for this object.

   f. How much work was done by the force F in lifting the mass to the height h?

3. A nail partially driven into a board, is driven in further by dropping a mass onto it. The mass, m, is released a distance y_0 above the head of the nail and drives the nail an additional distance s into the board. Neglect the mass of the nail.

   a. Defining the zero of potential at the top of the nail, find the potential energy of the system (take the mass m as the system) at the instant m is released.

   b. What is the kinetic energy at that instant?

   c. Find the potential energy after the mass has driven the nail the additional distance s into the board.

   d. What is the kinetic energy at that instant?

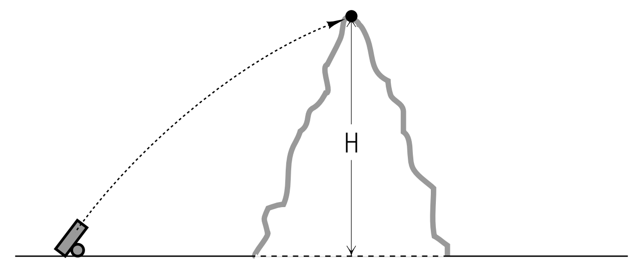
   e. Write the conservation of energy relation for this problem.

   f. How much work is done on the mass by the nail?

   g. How much work is done on the nail by the mass?

   h. Assuming that the nail exerts a constant force on the mass during the interval that the nail is slowing the mass to a stop, determine the magnitude of this force, F_N.

4.



An artillery shell fired from a gun at ground level leaves the muzzle of the gun with a speed of 50 m/s. The shell strikes a direct hit at a site on the top of a hill which is at an altitude of 45 m above ground level.

   a. Assuming no friction, use energy conservation to find the speed with which the shell hits at the site.

   b. If the shell strikes the ground at the site with a speed of 38 meters per second, what percentage of its original energy was lost (to friction) by the shell in its flight?

Brief Answers:

1. See this module's *text*.

2. See Problem 20 in this module's Problem Supplement.

3. See Problem 21 in this module's Problem Supplement.

4. See Problem 22 in this module's Problem Supplement.