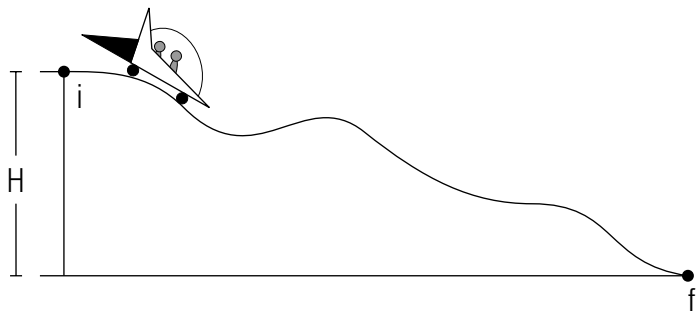


WORK, POWER, KINETIC ENERGY



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by

John S. Ross, Rollins College

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Input Skills:

1. Define the integral, evaluate integrals of polynomials (MISN-0-1).
2. Define the scalar product of two vectors and express it in component form (MISN-0-2).
3. Solve problems involving Newton's second law (MISN-0-16).

Output Skills (Knowledge):

- K1. Vocabulary: watt.
- K2. State the line integral definition of the work done by a force and explain how it reduces to other mathematical formulations for special cases.
- K3. Define the power developed by an agent exerting a force.
- K4. Derive the Work-Kinetic Energy Relation using Newton's second law and the work done by a variable force.
- K5. Define the kinetic energy of a particle.

Output Skills (Problem Solving):

- S1. Calculate the work done on an object given either:
 - a. one or more constant forces, or
 - b. a force that is a function of position along a prescribed path.
- S2. Use the definition of power to solve problems involving agents exerting constant forces on objects moving with constant velocity.
- S3. Use the Work-Kinetic Energy Relation to solve problems involving the motion of particles.

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“It’s love that makes the world go round.”
- Ancient Ditty

“Energy makes the world go round.
ENERGY explains EVERYTHING.”
- Modern Ditty to be
found in science books

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1. Introduction

The fundamental problem of particle dynamics is to determine the external forces that act on an object, then use them to find the position of the object as a function of time. In more detail, once we know the forces we add them to get the resultant force \vec{F}_R , which we put into Newton’s second law in order to get the acceleration \vec{a} . We can then find the final velocity by integrating the time-varying vector equation, inserting the initial velocity as an integration constant. We can repeat the integration to find the time-varying position. If the resultant force is constant in time these integrations produce $\vec{v} = \vec{v}_0 + \vec{a}t$ and $\vec{s} = \vec{v}_0t + (1/2)\vec{a}t^2$. However, there is an important class of problems in physics in which the force is not constant but varies as a function of the position of the particle. The gravitational force and the force exerted by a stretched spring are examples.

With the introduction of work, power, and energy, we have alternative methods for the solution of dynamics problems, methods that involve scalar equations rather than the vector equations required in the direct application of Newton’s laws.

More important than the alternative methods themselves is the concept of energy and the conservation law associated with it. The principle of conservation of energy is universal: it holds in all cases if all energy is carefully accounted for. It is true even for areas of physics where Newton’s laws are not valid, as in the atomic-molecular-nuclear world. It is of major interest in an energy-conscious world.

2. Work

2a. Meanings Associated with Work. Ever since your childhood you have heard and used the term “work.” Family members went to work. You were encouraged to work hard in school. You worked on your car, or it was difficult work riding a bike up the hill. All of us have an intuitive feeling about what is meant by work. However, it is necessary for the scientist/technologist to have a precise definition for meaningful communication with other professionals.

Technically, “work” is the amount of energy transferred into or out of a definite mechanical system through the action of a mechanical force acting on that system along a finite trajectory. By conservation of energy, work done on a system enhances its energy while work done by a system depletes it. If we can calculate this energy, we can often use it to then calculate what happens to various properties of the system.

2b. Definition for Constant Effective Force. We start with the special case where the effective force acting on an object is constant:

Work (for a constant effective force): the product of the *effective* force acting on an object times the path-wise displacement of the point of application of the force.

By “effective force” we mean the component of the force in the direction of the displacement: it is that portion of the force that is effective in doing work on the object. The “pathwise displacement” is the distance along its path that the point of application of the force moves while the force is being applied.

In Fig. 1, θ is the angle between the actual applied force \vec{F} and the horizontal displacement vector \vec{s} (not shown). The effective force is $|\vec{F}|\cos\theta = F\cos\theta$. This implies that in order for work to be done: (a) a force must act upon an object; (b) the point of application of the force must move through a displacement; and (c) the force must not be perpendicular to the displacement. Unless these conditions are fulfilled, no *technical* work has been done. Just thinking about this definition, or holding this page to read it, is not scientific “work.” However, if you take your pencil and copy the definition, you will be doing “work” in the technical sense of the word.

2c. Work Done by a Constant Force. By our definition, the work done by a constant force \vec{F} that makes a constant angle θ with the direc-

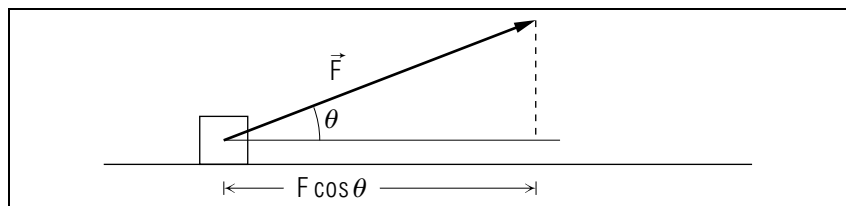


Figure 1. The *effective force* for horizontal displacement.

tion of the displacement \vec{s} (see Fig. 1) is:¹

$$W = |\vec{F}| \cos \theta |\vec{s}| = F s \cos \theta. \quad (1)$$

We recognize that the right hand side of this equation has the same form as the scalar (or dot) product of the two vectors \vec{F} and \vec{s} , so we can express the work done as:

$$W = |\vec{F}| |\vec{s}| \cos \theta = \vec{F} \cdot \vec{s}. \quad (2)$$

Work is a scalar quantity, although the force and displacement involved in its definition are vector quantities.

Notice that we can write Eq.(1) either as $(F \cos \theta) \times s$ or as $F \times (s \cos \theta)$. This suggests that the work can be calculated in two different ways: either we multiply the magnitude of the displacement by the component of the force in the direction of the displacement or we multiply the magnitude of the force by the component of the displacement in the direction of the force. These two ways are entirely equivalent.

Work can be positive or negative since $\cos \theta$ can take on positive or negative values ($-1 \leq \cos \theta \leq +1$). If the force acts in the displacement direction, the work is positive. If the force acts in the opposite direction to the displacement, the work is negative. For example, consider a person lowering an object to the floor. In this case \vec{F} points up and \vec{s} points down. While lowering the object, negative work is done by the upward force of the person's hand.

2d. Units of Work. The units of work are products of units of force and units of distance. In SI units, work is expressed in joules, abbreviated J. A joule is a newton-meter: one joule is the work done by a force of one newton when it moves a particle one meter in the same direction as the force. Recalling that $\text{N} = \text{kg m/s}^2$ we have that:

$$\text{J} = \text{N m} = \text{kg m}^2/\text{s}^2.$$

¹See "Vectors I: Products of Vectors" (MISN-0-2).

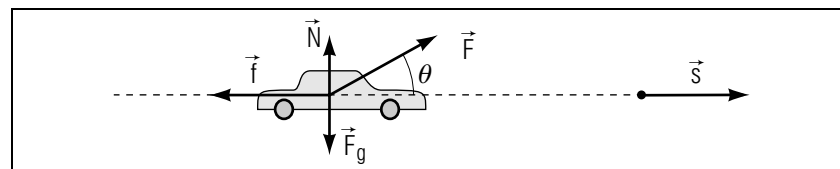


Figure 2. One-body force diagram for a pulled car (see text).

The name joule was chosen to honor James Joule (1816-1889), a British scientist famous for his research on the concepts of heat and energy.

2e. Illustration of the Work Concept. An Example/Problem: A Toyota (mass equal to $1.0 \times 10^3 \text{ kg}$) that had run out of gas was pulled down a level street by 3 people, each exerting $7.0 \times 10^2 \text{ N}$ of force on a rope inclined at 30.0° to the horizontal. The motion was at constant velocity because of friction, mostly between the tires and the street. The people pulled the car for one block (150 meters) before becoming tired and quitting.

- How much work was done by the people?

$$\begin{aligned} F_{\text{people}} &= \sum_{i=1}^3 F_i = 3(700 \text{ N}) = 2100 \text{ N} \\ W_{\text{people}} &= \vec{F} \cdot \vec{s} = F(\cos \theta) s = (2100 \text{ N})(\cos 30^\circ)(150 \text{ m}) \\ &= 2.7 \times 10^5 \text{ J.} \quad (2 \text{ digits of accuracy}) \end{aligned}$$

- How much work was done by the normal force \vec{N} ?

$$\text{Since } \sum \vec{F}_{\text{vertical}} = 0, \text{ then } \vec{N} = (mg - F \sin \theta) \hat{y}.$$

$$W_{\text{normal}} = \vec{N} \cdot \vec{s} = 0, \text{ since } \vec{N} \perp \vec{s}.$$

- How much work was done by the gravitational force, \vec{F}_g ?

$$\text{The answer: } W_{\text{gravity}} = \vec{F} \cdot \vec{s} \text{ which is also zero, since } \vec{F}_g \perp \vec{s}.$$

These last two cases emphasize that, whenever $\theta = 90^\circ$, the work done will be zero.

- Where did the energy go? We found that 2.7×10^5 joules of work were performed, by the people, on the car-earth system. This means that the people lost, and the car-earth system gained, 2.7×10^5 joules

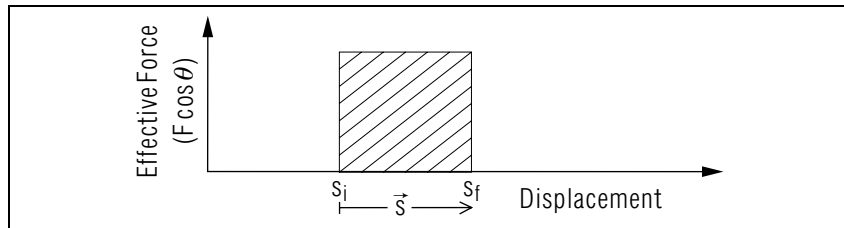


Figure 3. The effective force versus the displacement.

of energy. What happened to that energy? Some might have gone into energy of motion, kinetic energy of the car, but the car wound up not having any motion. In fact, the energy went into heating the pavement, the tires, the axles, the wheel bearings, the wheel bearing grease, and eventually the air surrounding these items as they cooled off. Finally, the atmosphere radiated some of the energy out into space and it became lost to the earth.

2f. Graphical Interpretation of Work. For a graphical interpretation of the work concept we plot the effective force ($F \cos \theta$) versus the displacement during the interval from the initial position s_i to the final position s_f . The work done is $(F \cos \theta)(s)$, the area under the curve shown in Fig. 3.

3. Work Done by Variable Forces

3a. Work During Infinitesimal Displacement. Let us now consider the more usual case where the work is done by a force whose value will depend on the position of the point of application. For a force that is changing only in magnitude, we can represent the situation graphically as in Fig. 4.

In order to find the work done for some displacement, we imagine dividing the displacement into a very large number of infinitesimal intervals. The work done by a force $\vec{F}(s)$ during any one infinitesimal displacement $d\vec{s}$ is given by:

$$dW = \vec{F}(s) \cdot d\vec{s}. \quad (3)$$

In order to obtain the total work done, which is a finite measurable parameter, we must sum up (integrate) those (infinitesimal) increments of work.

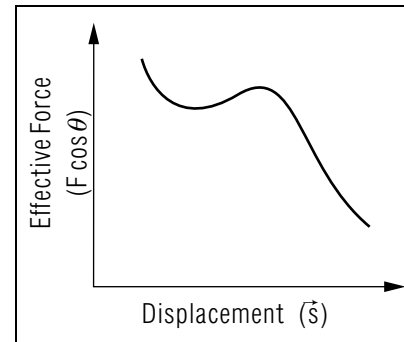


Figure 4. A variable force versus displacement.

3b. One Dimensional Motion: An Integral. As a first stab at integrating Eq. (3), let us investigate the situation where the force and the displacement are along the same line of action (say the x -axis) and the force is a known function of the position x . That is, $\vec{F} = F(x)\hat{x}$ where $F(x)$ is known. During a small displacement dx , so $d\vec{s} = dx\hat{x}$, the force does an amount of work dW given by:

$$dW = \vec{F} \cdot d\vec{s} = F(x) dx.$$

To obtain the work over a finite interval we sum these infinitesimal contributions by integrating. As the force moves the particle from a to b the work varies from zero to its final value:

$$\int_0^W dW = \int_a^b F(x) dx$$

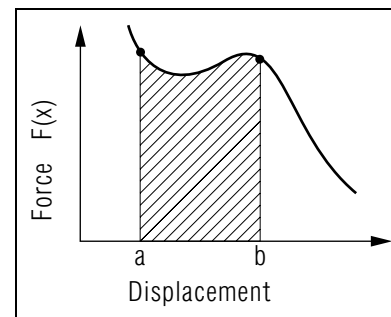


Figure 5. The area under the curve represents Work.

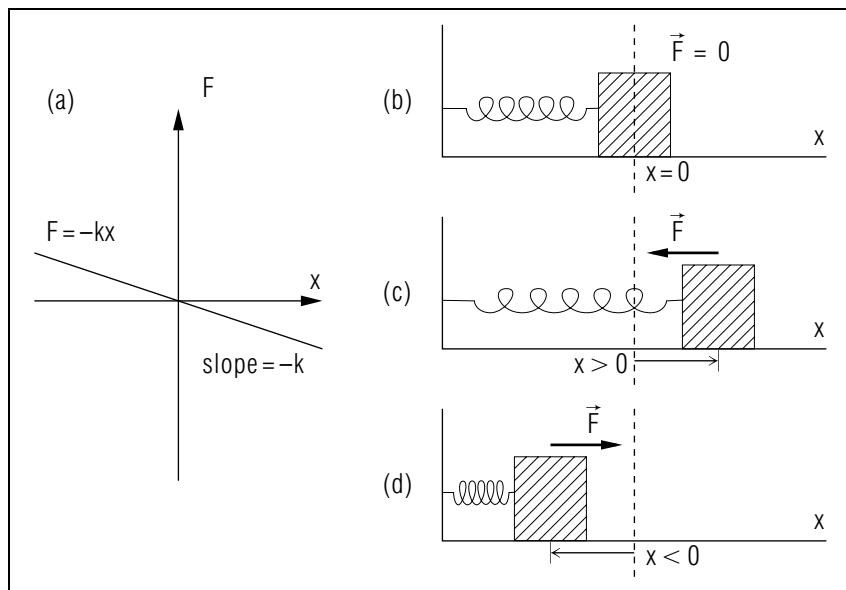


Figure 6. Force characteristics of a spring. (a) The spring force F as a function of its displacement x . (b) The spring in its equilibrium state. (c) The spring stretched by a displacement x to the right and with a spring force F to the left. (d) The spring compressed with a displacement x to the left and with the spring force F to the right.

or

$$W_{a \rightarrow b} = \int_a^b F(x) dx. \quad (4)$$

Graphically, the work is the area under the curve of $F(x)$ versus x (see Fig. 5). In order to calculate it we did not need to know the actual details of the motion, such as velocity as a function of time. Note that the graphical representation illustrates that work requires a displacement (if we are to have an area under the curve) and the notation $W_{a \rightarrow b}$ also serves to remind us of this fact. Work is not

a function of a single position in space like $F(x)$. Work at a point has no meaning; only over a displacement is it meaningful.

3c. Example: a Stretched Spring. As a helical spring is stretched or compressed, away from its equilibrium position, the spring resists with a force that is fairly accurately linear (unless it is poorly made or becomes

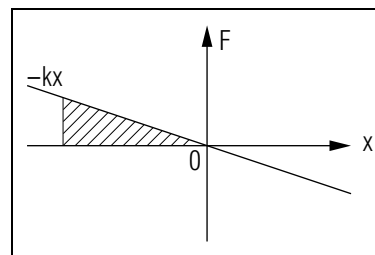


Figure 7. The work done by a force compressing a spring from 0 to x is the area $(1/2)kx^2$ under the “force versus displacement” curve.

deformed). That is, if a “linear” spring is stretched or compressed a distance x , it resists with a force $F = -kx$. Here the $(-)$ sign indicates that the spring’s force is opposite to the direction of the displacement from equilibrium. We call this a “restoring” force since it tends to restore the spring to its equilibrium position. The quantity k is called the “spring constant”: it is a measure of the “stiffness” of the spring. By the way, saying the spring is in its “equilibrium position” merely means that it is neither compressed nor stretched.

Suppose one finds that a 36 N force compressed a particular spring by 6.0 cm. How much work is done by the force if it compresses the spring by 5.0 cm? First, note that the value of the spring constant is:

$$k = -\frac{F}{x} = \frac{-36 \text{ N}}{-6.0 \times 10^{-2} \text{ m}} = 6.0 \times 10^2 \text{ N/m}.$$

Then the work done by the compressing force is:

$$W = \int \vec{F}_R \cdot d\vec{s} = \int_{-x}^0 (-kx) dx,$$

since \vec{F} is parallel to $d\vec{s}$ and is in the same direction. Thus:

$$W = -k \int_{-x}^0 x dx = \frac{kx^2}{2} = 0.75 \text{ J}.$$

This transfer of energy depleted the energy of the system applying the force and increased the (internal) energy of the spring.

We can arrive at the same result graphically by calculating the area under the “ F versus x ” curve. Since the area of a triangle is half its height times its base, we have:

$$\frac{1}{2}(-kx)(-x) = \frac{1}{2}kx^2.$$

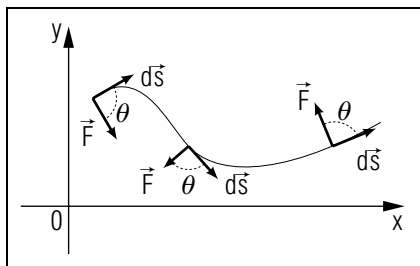


Figure 8. A force whose point of application follows a curved path.

3d. General Motion: A Line Integral. The force \vec{F} doing work may vary in direction as well as in magnitude, and the point of application may move along a curved path. To compute the work done in this general case we again divide the path up into a large number of small displacements $d\vec{s}$, each pointing along the path in the direction of motion. At each point, $d\vec{s}$ is in the direction of motion.

The amount of work done during a displacement $d\vec{s}$ is:

$$dW = \vec{F}(s) \cdot d\vec{s} = F(s)(\cos \theta) ds,$$

where $F(s)\cos\theta$ is the component of the force along the tangent to the trajectory at $d\vec{s}$. The total work done in moving from point s_i to point s_f is the sum of all the work done during successive infinitesimal displacements:

$$W = \vec{F}_1 \cdot d\vec{s}_1 + \vec{F}_2 \cdot d\vec{s}_2 + \vec{F}_3 \cdot d\vec{s}_3 + \dots$$

Replacing the sum over the line segments by an integral, the work is found to be:

$$W_{A \rightarrow B} = \int_A^B \vec{F}(s) \cdot d\vec{s} = \int_A^B F(s)(\cos \theta) ds, \quad (5)$$

where θ is a function of s , the position along the trajectory. This is the most general definition of the work done by a force $\vec{F}(s)$. We cannot evaluate this integral until we know how both F and θ vary from point to point along the path.

For any vector \vec{V} which is a function of position, an integral of the form

$$\int_i^f \vec{V} \cdot d\vec{s},$$

along some path joining points i and f , is called “the line integral of \vec{V} .” Equation (5) is of this nature because it is evaluated along the actual path in space followed by the particle as it moves from i to f .

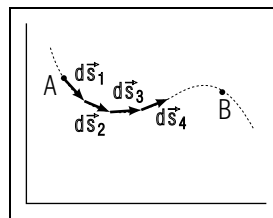


Figure 9. The total work is the sum over successive infinitesimal displacements.

Equation (5) is the “line integral definition of work.” For each increment of displacement $d\vec{s}$ along the path, the corresponding increment of work $dW = \vec{F} \cdot d\vec{s}$ is calculated and then these scalar quantities are simply summed to give the work along the total path.

We can obtain an equivalent general expression for Eq. (5) by expressing \vec{F} and $d\vec{s}$ in scalar component form. With $\vec{F} = F_x\hat{x} + F_y\hat{y} + F_z\hat{z}$ and $d\vec{s} = dx\hat{x} + dy\hat{y} + dz\hat{z}$, the resulting work done in going from position $A = (x_A, y_A, z_A)$ to position $B = (x_B, y_B, z_B)$ can be expressed as:

$$W_{i \rightarrow f} = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz, \quad (6)$$

where each integral must be evaluated along a projection of the path.

Example: At Space Mountain in Disney World the space rocket ride is simulated by a cart which slides along a roller coaster track (see cover) which we will consider to be frictionless. Starting at an initial position #1 at height H above the ground, find the work done on this rocket when you ride it to the final position f at the bottom of the track (see Fig. 10).

The forces that act on this “rocket” are: the force of gravity, $\vec{F}_g = -mg\hat{y}$ and the “normal” surface reaction force \vec{N} . The total work done by the resultant force $\vec{F}_R = \vec{F}_g + \vec{N}$ on the rocket between the two end points is:

$$W_{i \rightarrow f} = \int_i^f \vec{F}_R \cdot d\vec{s} = \int (\vec{F}_g + \vec{N}) \cdot d\vec{s} = \int \vec{F}_g \cdot d\vec{s} + \int \vec{N} \cdot d\vec{s}.$$

Since the surface force is always perpendicular to the path, it does no work. That is, $\int \vec{N} \cdot d\vec{s} = \int N \cos \phi ds = 0$, since the angle between \vec{N} and $d\vec{s}$ is always 90° . This is true despite the fact that the angle between \vec{F}_g and $d\vec{s}$ changes continuously as the rocket goes down the track.

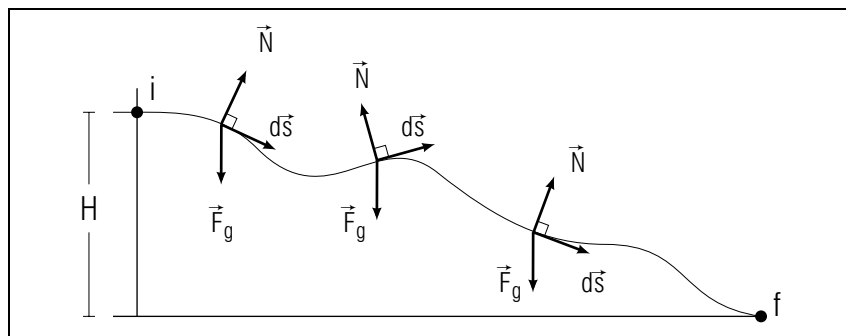


Figure 10. Force diagram for rocket cart.

Now we can write $d\vec{s} = dx\hat{x} + dy\hat{y}$ so that:

$$\begin{aligned} W_{i \rightarrow f} &= \int \vec{F}_g \cdot d\vec{s} = \int (-mg\hat{y}) \cdot (dx\hat{x} + dy\hat{y}) \\ &= - \int_H^0 mg dy = -mg \int_H^0 dy = mgH \end{aligned}$$

Here we see that the line integral reduces to a simple summation of the elements dy , which are the projections of $d\vec{s}$ on the constant direction \vec{F}_g . The answer, mgH , is an important one to remember. It is the energy of the earth-plus-rocket system that was transferred from gravitational energy to mechanical energy. That answer holds for any earth-plus-object system.

4. Power

4a. Definition of Power. Let us now consider the time involved in doing work. The same amount of work is done in raising a given body through a given height whether it takes one second or one year to do so. However, the rate at which work is done is often as interesting to us as is the total work performed. When an engineer designs a machine it is usually the time rate at which the machine can do work that matters.

Instantaneous power is defined as the time rate at which work is being done at some instant of time. That is, it is the limit, as the time interval approaches zero, of the amount of work done during the interval divided by the interval. Since this is the definition of the time derivative, we have:

$$P = \frac{dW}{dt}. \quad (7)$$

For constant force or velocity, using Eq. (3) and $\vec{v} = d\vec{s}/dt$ we get:

$$P = \vec{F}_{\text{const}} \cdot \vec{v} \quad ; \quad P = \vec{F} \cdot \vec{v}_{\text{const}}. \quad (8)$$

The average power P_{av} during a time interval Δt is:

$$P_{av} = \frac{W}{\Delta t}.$$

If the power is constant in time, then $P_{\text{const}} = P_{av}$ and

$$W = P_{\text{const}} \Delta t. \quad (9)$$

4b. Units of Power. According to the definition of power, its units are units of work divided by units of time. In the MKS system, the unit of power is called a watt, abbreviated W, which is equivalent to a joule per second. One watt is the power of a machine that does work at the rate of one joule every second. Recalling that $J = m^2 \text{ kg/s}^2$, we have that:

$$W = J/s = m^2 \text{ kg/s}^3.$$

The name watt was chosen in honor of the British engineer James Watt (1736-1819) who improved the steam engine with his inventions.

Work can be expressed in units of power \times time. This is the origin of the term kilowatt-hour (kWh). One kilowatt-hour is the work done in 1 hour by an agent working at a constant rate of 1 kW (1000 W). Electricity is sold “per kWh.”

Example: Under very intense physical activity the total power output of the heart may be 15 unitwatts. How much work does the heart do in one minute at this rate?

$$W = P_{\text{const}} \Delta t = (15 \text{ W}) (60 \text{ s}) = 900 \text{ J}.$$

5. Kinetic Energy

5a. Definition of Kinetic Energy. A particle’s kinetic energy is defined as the amount of energy a particle has, due solely to its velocity. Using Newton’s second law and the definition of total work, one can show that the kinetic energy, E_k , of a particle of mass m traveling at velocity v is:

$$\text{KINETIC ENERGY} = E_k = \frac{1}{2}mv^2. \quad (10)$$

This is valid whenever Newtonian mechanics is valid. We can easily illustrate this derivation for the special case of a resultant force, \vec{F}_R , that acts on a particle of mass m along the direction of its displacement. For this case the total work done on the particle is:

$$W_{T,i \rightarrow f} = \int_{s_i}^{s_f} \vec{F}_R \cdot d\vec{s} = \int_{x_i}^{x_f} F_R dx. \quad (11)$$

Since the force, hence the acceleration, is along the direction of displacement, we can use Eq. (11) and Newton's second law² to write:

$$\begin{aligned} W_{i \rightarrow f} &= \int_{x_i}^{x_f} m a dx = \int_{x_i}^{x_f} m \frac{dv}{dt} dx = \int_{x_i}^{x_f} m dv \frac{dx}{dt} \\ &= \int_{x_i}^{x_f} m v dv = m \int_{x_i}^{x_f} v dv = \frac{mv_f^2}{2} - \frac{mv_i^2}{2}. \end{aligned} \quad (12)$$

Thus the work that went into accelerating the particle, from v_i to v_f , is exactly equal to the change in the kinetic energy for that particle. Note that kinetic energy is a scalar quantity. It is, as we shall see, just as significant as the vector quantity, momentum, that is also a quantity used to describe a particle in motion.

From its definition, kinetic energy has dimensions ML^2/T^2 , either mass multiplied by the square of speed, or, since it is equivalent to work [as shown in Eq. (12)], force multiplied by distance. Thus, the kinetic energy of a particle may be expressed in *joules*. It follows that a 2 kg particle moving at 1 m/s has a kinetic energy of 1 J.

The kinetic energy of a particle can be expressed in terms of the magnitude of its linear momentum, $m\vec{v} = \vec{p}$:

$$E_k = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m} = \frac{|\vec{p}|^2}{2m} \quad (13)$$

Example: A particle of mass m starts from rest and falls a vertical distance h . What is the work done on this particle and what is its final kinetic energy?

The particle experiences only a single constant force, $\vec{F} = -mg\hat{y}$, in the downward direction. The displacement, $d\vec{s} = -dy\hat{y}$, is also downward

²See "Free-Body Force Diagrams, Frictional Forces, Newton's Second Law" (MISN-0-15).

for a distance h . Therefore the work done by the gravitational force is:

$$W = \int \vec{F} \cdot d\vec{s} = \int_0^h (-mg\hat{y}) \cdot (-dy\hat{y}) = \int_0^h mg dy = mgh.$$

Since this is the only work done on the particle, it is equal to the final kinetic energy:

$$mgh = \frac{1}{2}mv^2.$$

Solving for velocity:

$$v = \sqrt{2gh},$$

which is the same result we obtain from kinematics for an object falling with constant acceleration g (a "freely-falling" object).

Example: It is possible for a person with a mass of 7.0×10^1 kg to fall to the ground from a height of 10.0 meters without sustaining an injury. What is the kinetic energy of such a person just before hitting the ground? The velocity of the person is:

$$v = \sqrt{2gh} = \sqrt{2(9.8 \text{ m/s}^2)(10 \text{ m})} = 14 \text{ m/s}.$$

Then the kinetic energy of the person is:

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}(70 \text{ kg})(14 \text{ m/s})^2 = 6.9 \times 10^3 \text{ J}.$$

5b. The Energy Concept. Of all the concepts of physics, that of energy is perhaps the most far-reaching. Everyone, whether a scientist or not, has an awareness of energy and what it means. Energy is what we have to pay for in order to get things done. The work itself may remain in the background, but we recognize that each gallon of gasoline, each Btu of heating gas, each kilowatt-hour of electricity, each calorie of food value, represents, in one way or another, the wherewithal for doing something. We do not think in terms of paying for force, or acceleration, or momentum. Energy is the universal currency that exists in apparently countless denominations, and physical processes represent a conversion from one denomination to another.

The above remarks do not really define energy. No matter. It is worth recalling the opinion of H. A. Kramers: "The most important and most fruitful concepts are those to which it is impossible to attach a well-defined meaning." The clue to the immense value of energy as a concept lies in its transformation. We often find energy defined in general as the ability to do

work. A system possesses energy; it can do work. At any instant a system has a certain energy content. Part or all of this energy can be transformed into the activity of work. Work is only an active measure of energy and not a form of energy itself. Work is best regarded as a mode of transfer of energy from one form to another. It is a medium of exchange. In this unit we are dealing with only one category of energy—the kinetic energy associated with the motion of an object. If energy should be transferred from this form into chemical energy, radiation, or the random molecular and atomic motion we call heat, then from the standpoint of mechanics it is gone. This is a very important feature, because it means that, if we restrict our attention purely to mechanics, conservation of energy does not hold. Nevertheless, as we shall see, there are many physical situations in which total mechanical energy is conserved, and in such contexts it is of enormous value in the analysis of real problems.

6. The Work-Kinetic Energy Relation

6a. Derivation of the Relation. Equation (12) shows that the work done on a particle, when the resultant force acting on the particle is in the direction of the displacement, is equal to the change in the particle's kinetic energy. For the more general case of a force that is not in the direction of motion, we can still derive this valuable relation between the work done and the change in kinetic energy.

Write \vec{F}_R for the resultant force acting upon a particle of mass m , moving along a path between the two positions s_i and s_f , then:

$$\begin{aligned}\sum W_{i \rightarrow f} &= \int_{s_i}^{s_f} \vec{F}_R \cdot d\vec{s} = \int m\vec{a} \cdot d\vec{s} = m \int \frac{d\vec{v}}{dt} \cdot d\vec{s} \\ &= m \int d\vec{v} \cdot \frac{d\vec{s}}{dt} = m \int d\vec{v} \cdot \vec{v}.\end{aligned}$$

Thus:

$$\sum W_{i \rightarrow f} = m \int_{v_i}^{v_f} d\vec{v} \cdot \vec{v} = \frac{mv_f^2}{2} - \frac{mv_i^2}{2} = E_{k,f} - E_{k,i} = \Delta E_k.$$

The evaluation of the integral in this last step can be easily seen if you write \vec{v} and $d\vec{v}$ in scalar component notation.³ In summary,

$$W_{i \rightarrow f} = \int_i^f \vec{F}_R \cdot d\vec{s} = \Delta E_k. \quad (14)$$

Equation (14) is known as the Work-Kinetic Energy Relation and it is valid no matter what the nature of the force:

The total work done on a particle (by the resultant force acting on it), between some starting point s_i and ending point s_f , is the change in the particle's kinetic energy between those two end points.

This relation applies quite apart from the particular path followed, so long as the total work done on the particle is properly computed from the resultant force.

6b. Significance of the Relation. The work-kinetic energy relation is not a new independent relationship of classical physics. We have derived it directly from Newton's second law, utilizing the definitions of work and kinetic energy. This relation is helpful in solving problems where the work done by the resultant force is easily computed, or where we are interested in finding the speed of a particle at a particular position. However, we should recognize that the work-kinetic energy principle is the starting point for formulating some sweeping generalizations in physics. We have stressed that this principle can be applied when ΣW is interpreted as the work done by the resultant force acting on a particle. In many cases however, it is more useful to compute separately the work done by each of certain types of forces which may be acting and to give these special designations. This leads us to the identification of different types of energy, and the principle of the conservation of energy.⁴

Example: (a) How much work is required to stop a 1.0×10^3 kg car that

³For additional discussion of the mathematical steps presented in this derivation see *Newtonian Mechanics*, A. P. French, W. W. Norton & Co. (1971), pp. 368-72, or *Physics for Scientists and Engineers, Volume 1*, Melissinos and Lohkowitz, W. B. Saunders Company (1975), p. 170.

⁴See "Potential Energy, Conservative Forces, the Law of Conservation of Energy" (MISN-0-21).

is moving at a speed of 20.0 m/s?

$$\begin{aligned} W_{T,i \rightarrow f} &= E_{k,f} - E_{k,i} = 0 - \frac{1}{2}mv_i^2 = 0 - \frac{1}{2}(1000 \text{ kg})(20 \text{ m/s})^2 \\ &= -2.0 \times 10^5 \text{ J.} \end{aligned}$$

The negative (−) sign indicates that there was a decrease in the kinetic energy of the car; that is, work was done on the car by a braking force opposing its motion.

(b) If this car requires 100 meters to come to rest, what is the total resultant braking force acting on the car?

$$W_{i \rightarrow f} = \int_i^f \vec{F}_R \cdot d\vec{s} = -f \int_{s_i}^{s_f} ds,$$

since the total resultant force can be represented by a single constant braking force f which directly opposes the motion. Thus:

$$-2 \times 10^5 \text{ J} = -f(100 \text{ m}) \quad \text{or} \quad f = 2 \times 10^3 \text{ N.}$$

Example: In a previous example we found that a stretched spring did an amount of work $kx^2/2$ on an object when the spring returned to its equilibrium position from a displacement x .

We can now use the Work-Kinetic Energy relation to easily find the velocity of the block as it passes the equilibrium position:

$$\begin{aligned} W_{i \rightarrow f} &= \Delta E_k \\ \frac{kx^2}{2} &= \frac{mv_f^2}{2} - \frac{mv_i^2}{2}. \end{aligned}$$

Since it initially started from rest we find:

$$v_f = x \sqrt{k/m}.$$

Acknowledgments

Professor James Linneman made several helpful suggestions. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

PROBLEM SUPPLEMENT

Note: If you have trouble with a problem, see the activities in the *Special Assistance Supplement*. If there is a *help* reference, it also is in the *Special Assistance Supplement*.

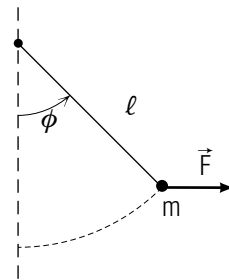
In all problems involving the calculation of *work*, you should start with the line-integral definition and show how it can be applied and simplified for the particular situation.

Take $g = 9.8 \text{ m/s}^2$, 1 HP = 550 ft lb/sec = 746 watts.

Problems 21, 22, and 23 also occur in this module's *Model Exam*.

- A crate whose mass is 39 kg is dragged at constant speed across the floor for 9.0 meters by applying a force of 100.0 N at an upward angle of 35 degrees to the horizontal.
 - What is the work done by the applied force *Answer: 5 Help: [S-1]*
 - What is the coefficient of friction between the crate and the floor? *Answer: 14 Help: [S-1]*
- A constant force $\vec{F} = (5.0\hat{x} + 2.0\hat{y} + 2.0\hat{z}) \text{ N}$ acts on an object during the displacement $\Delta\vec{r} = (10.0\hat{x} + 5.0\hat{y} - 6.0\hat{z}) \text{ m}$. Determine the net work done by this force during the displacement. *Answer: 6 Help: [S-2]*
- Determine the speed of a proton that has: kinetic energy = $8.0 \times 10^{-13} \text{ J}$ and mass = $1.67 \times 10^{-27} \text{ kg}$. *Answer: 13 Help: [S-3]*
- What is the kinetic energy of a 5.0 kg object with a velocity: $\vec{v} = (3.0\hat{x} + 5.0\hat{y}) \text{ m/s}$? *Answer: 3 Help: [S-4]*
- An escalator in a department store joins two floors that are 5.0 meters apart.
 - When a 51 kg woman rides up this escalator, how much work does the motor do to lift her? *Answer: 18 Help: [S-5]*
 - How much power must the motor develop in order to carry 80 people between floors in one minute, if the average person's mass is 70.2 kg? *Answer: 23 Help: [S-5]*

6. What horsepower must be developed by the engine of a racing car when a forward thrust of 2.8×10^3 N moves it at a constant velocity of 67.5 m/s (150 mph)? *Answer: 2 Help: [S-6]*
7. A car with a mass of 1.2×10^3 kg is pulled at constant velocity up a sloping street, inclined at a 31 degree angle from the horizontal, by a truck having a tow cable attached at an angle of 41° to the street. The coefficient of friction between the car and the street is 0.30. What is the minimum work the tow truck would have to do on the car to move it 204 meters along the sloping street? *Answer: 12 Help: [S-7]*
8. A particle of mass m hangs from a string of length ℓ as shown. A variable horizontal force \vec{F} starts at zero and gradually increases, pulling the particle up very slowly (equilibrium exists at all times) until the string makes an angle ϕ with the vertical. Calculate the work done by \vec{F} to raise m to this position. *Answer: 1 Help: [S-8]*



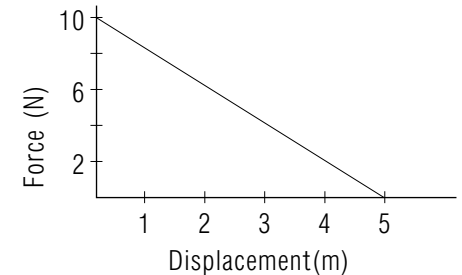
9. A particle is moving with a velocity v_0 along the x -axis at time t_0 . It is acted upon by a constant force in the x -direction until time t_1 . Show that:

$$\int_{t_0}^{t_1} (\vec{F} \cdot \vec{v}) dt = \frac{\Delta(p^2)}{2m}$$

where $\Delta(p^2)$ is the change in the square of its momentum. *Help: [S-9]*

10. A field goal kicker hits the ball at an angle of 42° with respect to the ground. The football, of mass 0.40 kg, sails through the air and hits the cross bar of the goal post 3.0×10^1 m away. If the cross bar is 3.0 meters above the ground, determine the work done by the gravitational force over the flight of the ball. *Answer: 11 Help: [S-10]*

11. A 2.0 kg particle, currently at the origin and having a velocity of $\vec{v} = 3.0\hat{x}$ m/s, is acted upon by the force shown in the graph.
- Determine the work done, both graphically and analytically, for a displacement from $x = 0.0$ to $x = 5.0$ meters. *Answer: 16 Help: [S-11]*
 - What is the velocity of the particle when it is at $x = 5.0$ m? *Answer: 24 Help: [S-11]*



12. A car whose mass is 1.8×10^3 kg has a safety bumper that can withstand a collision at 5.0 mph (2.25 m/s). Suppose the average retarding force of the energy absorbing bumper mechanism is 5.0×10^4 N. How much will the bumper be displaced if the car is going at 5.0 mph when it hits a tree? *Answer: 17 Help: [S-13]*
13. An object is thrown with a velocity of 61 m/s vertically downward from a height of 202 meters. What is its velocity when it hits the ground? Work this problem two ways—by the work-kinetic energy relation and by the laws of linear motion. *Answer: 9 Help: [S-14]*
14. A bullet pierces a 4.0 cm thick piece of metal armor plate with a velocity of 708 m/s and leaves the other side with a velocity of 310 m/s. Determine the thickness of the metal plate that would be required to stop the bullet completely. *Answer: 21 Help: [S-15]*
15. A Mercedes-Benz 450SL is moving at 41 m/s (90 mph) when the brakes become locked. How far will the car slide on:
- dry pavement ($\mu = 0.90$)? *Answer: 22 Help: [S-16]*
 - wet pavement ($\mu = 0.50$)? *Answer: 27 Help: [S-16]*
 - icy pavement ($\mu = 0.10$)? *Answer: 8 Help: [S-16]*
16. A girl in an archery class finds that the force required to pull the bowstring back is directly proportional to the distance pulled. She

finds that a force of 29 N is needed to pull the bow string back a distance of 0.10 meter. If she pulls a 55 gram arrow back a distance of 0.30 meters,

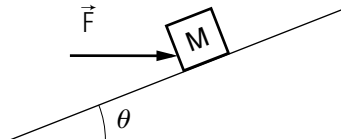
- what would be the velocity of the arrow if 81% of the work done was converted into kinetic energy? *Answer: 20 Help: [S-17]*
- If she shoots the arrow straight up, how high will it go (neglect the effects of air resistance)? *Answer: 26 Help: [S-17]*
- If the arrow actually only goes 91% of the height in part (b), what is the average resistive force of the air? *Answer: 29 Help: [S-17]*

17. A particle of mass 4.0 kg is acted on by the force:

$$\vec{F} = [(x + 2y)\hat{x} + (2y + 3xy)\hat{y}] \text{ N},$$

as the particle moves along a straight line path from the point (0,0) to the point (2.0 m, 2.0 m), that is along $x = y$.

- Find the work done by the force. *Answer: 19 Help: [S-18]*
 - If the particle was at rest when it was at the coordinate origin, what is its speed when it is at $x = 2.0 \text{ m}$, $y = 2.0 \text{ m}$? *Answer: 7 Help: [S-18]*
18. An airplane lands on an aircraft carrier, and is halted by the arresting cable. The force equation for the cable is $\vec{F} = -kx^2\hat{x}$, where $k = 1.20 \times 10^2 \text{ N/m}^2$. For an initial plane velocity of 81 m/s and a plane mass of $2.25 \times 10^3 \text{ kg}$, determine how far the plane will travel after being hooked. *Answer: 15 Help: [S-19]*
19. What horsepower engine would be required if you wish to pull a 54 kg woman water skier up from the water in 2.0 s with a final velocity of 13 m/s? The mass of the boat plus driver is 206 kg and that of the engine is 45.0 kg. During this period the average drag energy of the skier and boat is $5.0 \times 10^4 \text{ J}$. *Answer: 6 Help: [S-20]*
20. A constant horizontal force \vec{F} pushes an object of mass M up a frictionless incline which makes an angle θ with the horizontal.



- Starting with the object midway *up* the incline, moving the speed of v_0 directed up along the incline, use the work-kinetic energy

relation to find the velocity of the object when it's at a point a distance D further up the incline. *Answer: 31 Help: [S-21]*

- If at this same starting point as in part (a) the object started with the speed v_0 directed *down* along the incline (same forces acting as before), use the work-kinetic energy relation to find the velocity of the object when it's at the point a distance D further up the incline. *Answer: 33 Help: [S-21]*
 - Explain the relation between your answers to parts (a) and (b). What is the difference in the overall motion between the two cases? *Answer: 35 Help: [S-21]*
21. The gravitational force on an object of mass m which is at or above the surface of the earth, say at a total distance r from the center of the earth, has a magnitude $K m/r^2$ and is directed toward the center of the earth. Here K is a constant equal to $3.99 \times 10^{14} \text{ m}^3/\text{s}^2$. Hence the force is not constant but diminishes the farther the object is from the earth's center.
- At the surface of the earth, recall from previous knowledge that this force is mg . With this knowledge of the force at the surface of the earth and with K given above, determine the radius of the earth. *Answer: 32 Help: [S-22]*
 - What is the minimum amount of work that you must do (energy you must expend) in order to move an object of mass m radially outward from the surface of the earth to a distance r from the center of the earth ($r > R_e$)? *Help: [S-22]* (HINT: What's the *minimum* force you must exert? That determines the *minimum* work). *Answer: 30 Help: [S-22]*
 - From this result, calculate how much energy it takes to move a person of mass 65 kilograms from the surface of the earth to an infinite distance away (disregard gravity forces other than from the earth). *Answer: 36 Help: [S-22]*
22. Suppose a car engine is delivering $1.60 \times 10^2 \text{ hp}$ at a continuous rate, keeping the car at 25 m/s (56 mph). Determine the force the engine is overcoming. *Answer: 34 Help: [S-23]*

Brief Answers:

1. $mg\ell(1 - \cos \phi)$
2. 253 hp
3. 85 J
4. 48 J
5. 7.4×10^2 J
6. 51 hp.
7. 3.0 m/s
8. 8.6×10^2 m
9. 88 m/s
11. -11.8 J
12. 1.47×10^6 J
13. 3.1×10^7 m/s
14. $[F \cos \theta / (mg - F \sin \theta)] = 0.25$
15. 57 m
16. 25 J
17. 9.1×10^{-2} m
18. 2.5×10^3 J
19. 18 J
20. 2.0×10^1 m/s
21. 4.9×10^{-2} m
22. 95 m
23. 4.6×10^3 W or 6.1 hp
24. $5.8 \hat{x}$ m/s

26. 2.0×10^1 m
27. 0.17 km
29. 5.3×10^{-2} N
30. $Km \left[\frac{1}{R_e} - \frac{1}{r} \right]$.
31. $\left[\frac{2D}{M} (F \cos \theta - Mg \sin \theta) + v_0^2 \right]^{1/2}$.
32. 6.38×10^6 m
33. Same answer as (BM).
34. 1.1×10^3 lb
35. In the second case the object moves down, slowing, stops, turns around and when it gets to original point it has v_0 upward speed, and so on up to point distance D upward.
36. 4.06×10^9 J.

SPECIAL ASSISTANCE SUPPLEMENT

PURPOSE. If you have trouble with the Problem Supplement, carry out the activities in this Supplement.

CONTENTS.

1. Introduction
2. Understanding the Definitions
3. Advice and Problems With Hints Available
4. Answers for the Assistance Supplement
5. Hints for the *Problem and Assistance Supplements*

Sect. 1. Introduction. We apply these general relationships:

$$W_{A \rightarrow B} = \int_A^B \vec{F} \cdot d\vec{s} = \Delta E_k$$

and

$$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}.$$

You should fully understand those relationships so you can recognize their applications to special cases.

Sect. 2. Understanding the Definitions.

- a. Complete the following table, which summarizes the basic characteristics of the quantities in the first column:

	Vector or Scalar	Possible Sign +, 0, or -	SI Unit	SI Symbol	Unit in MKS
Force					
Displacement					
Work					
Power					

Answer: 49

- b. Symbol Recognition: write out in words the meaning of each of the following symbols:

- | | | |
|---------------------------|----------------|------------------|
| (a) $W_{A \rightarrow B}$ | (d) $d\vec{s}$ | (g) ΔE_k |
| (b) W_T | (e) E_k | (h) P_{av} |
| (c) \vec{F}_R | (f) dW | (i) $\vec{F}(s)$ |

Answer: 50

- c. Describe under what conditions each of the following is a valid expression for $\int_A^B \vec{F}(s) \cdot d\vec{s}$:

- | | |
|-------------------------------|-------------------------------|
| (a) $= \vec{F} \cdot \vec{s}$ | (d) $= F \cos \theta \int ds$ |
| (b) $= F s$ | (e) $= \int F \cos \theta ds$ |
| (c) $= F \int ds$ | |

Answer: 51

- d. Indicate whether each of the following expressions is either correctly or incorrectly written:

- | | |
|---|---|
| (a) $\int dW = \int F \cos \theta ds$ | (f) $W = \int \vec{F} d\vec{s}$ |
| (b) $W = \int F s$ | (g) $W = F s$ |
| (c) $\int dW = \int \vec{F} \cdot d\vec{s}$ | (h) $\int \vec{F} \cdot d\vec{s} = \vec{F} \cdot \vec{s}$ |
| (d) $dW = \vec{F} \cdot \vec{s}$ | (i) $W = \int P dt$ |
| (e) $W = F ds$ | (j) $P = \vec{F} \vec{v}$ |

Answer: 52

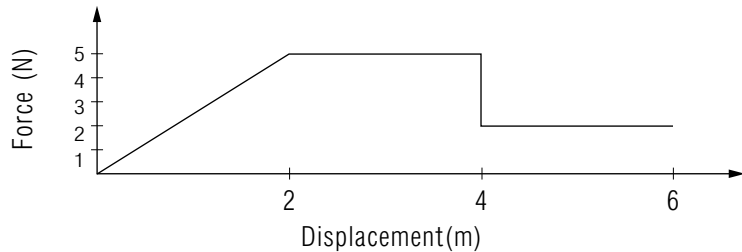
- e. How is it possible for an object which is moving to have: $\int d\vec{s} = 0$?
Answer: 53

- f. State the three cases for which: $W = \int \vec{F} \cdot d\vec{s} = 0$. *Answer: 54*

- g. Show that: $(F_x \hat{x} + F_y \hat{y} + F_z \hat{z}) \cdot (dx \hat{x} + dy \hat{y} + dz \hat{z})$ reduces to $F_x dx + F_y dy + F_z dz$. *Answer: 55*

- h. A 90 kg ice hockey player skating during warmup has a kinetic energy of 400 J. (a) If during the game he skates at three times his warmup speed what is his new kinetic energy? (b) If during the game he collides with another player, of the same mass, velocity, and slide along the ice together, what is their combined kinetic energy? *Answer: 56*

- i. Compute the average kinetic energy of a 70 kg sprinter who covers 100 m in 9.8 seconds. *Answer: 57*
- j. When we are moving at a constant velocity what can we say about ΔE_k ? *Answer: 58*
- k. If an object with forces acting on it is moving with a velocity v and $\Delta E_k = 0$, then why does $W = \int \vec{F} \cdot d\vec{s} = 0$? *Answer: 59*
- l. What do we mean by: (a) “+” work? (b) “-” work? (c) “+” power? (d) “-” power? *Answer: 60*
- m. Why is kinetic energy either 0 or + (that is, it is never negative)? *Answer: 61*
- n. Calculate both analytically and graphically the work done by the force shown below in moving a particle from $x = 0$ to $x = 6$ meters.



Answer: 62

- o. If a force $\vec{F} = (2x + 3x^3)\hat{x}$ N acts on an object, what is the work done by this force for a displacement from $x = 2$ to $x = 5$ meters? *Answer: 63*
- p. Starting with $W = \int_A^B \vec{F} \cdot d\vec{s}$, derive the expression:

$$W = \int_{t_1}^{t_2} P dt.$$

- q. Show that: $\int_{v_i}^{v_f} \vec{v} \cdot d\vec{v} = \frac{1}{2}(v_f^2 - v_i^2)$.

Hint – write the vectors in component form.

- r. (a) What power is developed by a 70 kg man when climbing, in 20 seconds, a flight of stairs that rises 12 meters?

- (b) What power must be provided to move a 10 kg block horizontally on a surface with friction, if a force of 1.0×10^2 N gives it a constant velocity of 2.0×10^1 m/s? *Answer: 64*
- s. Why is the definition of work a valuable concept? *Answer: 65*
- t. Why is the work-energy principle a useful relationship? *Answer: 66*

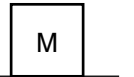
Sect. 3. Advice and Problems With Hints Available.

In working out the solution to a problem:

- List the properties or data given.
- Clearly indicate which quantities are to be found.
- Determine which conceptual relationship(s) is to be applied.
- Simplify the general relationship for the specific case.

Attempt to work these problems without assistance before consulting the answers or hints provided in Sections 5 and 6.

- An object which has a mass of 20 kg sits on a horizontal surface. It is found that the frictional force between the object and the surface when the object moves across the surface is a constant 15 N. Suppose that you move this object with a horizontal force a distance of 50 m at a constant speed of 3 m/s.



How much work is done by the resultant of all the forces acting on this object? How much work do you do? *Answer: 39*

- A quarterback throws a long pass and the football (mass = 4.0×10^2 gm) leaves his hand at an angle of 4.0×10^1 degrees to the horizontal ground. The ball hits the tight end, who lets it slip through his hands so that it falls to the ground.
 - What was the work done on the ball during the first 0.10 m of travel just after it left the quarterback's hand? This displacement may be considered infinitesimal compared to the total trajectory. *Answer: 43*

- b. What is the work done on the ball during an 0.10 m displacement at the peak of its trajectory? *Answer: 38*
- c. What is the increment of work done on the ball during an 0.10 m displacement as it falls vertically to the ground just after hitting the tight end? *Answer: 41*
3. A 1.00×10^2 kg fullback has a velocity of 9.0 m/s as he hits the line. If he is to be held to a 2.0 m gain, what average resultant force must be exerted on him by the opposing team? *Answer: 44*
4. A baseball (mass = 0.15 kg) has a kinetic energy of 5.0×10^1 J at the peak of its trajectory, which is 34 m above ground. What was the velocity of the ball just after it was hit by the bat? *Answer: 37*
5. A 1.00×10^3 kg elevator starts from rest and experiences a constant upward acceleration of 2.0 m/s^2 . Determine the power required to maintain this acceleration for 4.0 s. *Answer: 38*

Answers for the Assistance Supplement.

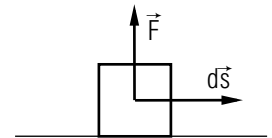
37. 36.5 m/s. The work done by gravity is $mgh = \Delta E_k$.
38. Zero, since $m\mathbf{g} \hat{y} \perp d\vec{s}$.
39. When you draw your one-body force diagram you should have four forces acting on this object: gravity, the normal force upward exerted by the surface, the force you apply, and the frictional force in the opposite direction. The key phrase in this problem description is "constant speed," which implies $\vec{a} = 0$ so $\vec{F}_R = 0$, hence $W_R = 0$.
- What force do you exert? (J) How much work do you do? (K)
40. 8000 W. Use $s = vt + at^2/2$ to find the distance travelled and evaluate $W = \int \vec{F} \cdot d\vec{s} = mas$, then solve for power.
41. +0.39 J. $W = \int mg(+1) ds$.
42. It does not change. This is consistent with $W = \Delta E_k = 0$, since the speed is constant.
43. -0.25 J, $W = \int mg(\cos 130^\circ) ds$. *Help: [S-39]*
44. 2025 N. Use the work-kinetic energy relation, assuming the opposing force starts acting at the line of scrimmage.
45. 6 meters. What is the kinetic energy of this object? (See L). Now apply the work-kinetic energy principle, where the only force acting on the block is the resistive frictional force.
46. Constant v so $\vec{F} = 0$ so you exert 15 N.
47. given v and d , $t = d/v$ so $W = Pt = (Fv) \times (d/v) = 750 \text{ J}$. Also: $W = F \times d = 750 \text{ J}$.
48. 90 J.
- 49.

	Vector or Scalar	Possible Sign	SI Unit	SI Symbol	Unit in MKS
Force	Vector	+, 0, -	newton	N	kg m/s^2
Displacement	Vector	+, 0, -	meter	m	m
Work	Scalar	+, 0, -	joule	J	$\text{kg m}^2/\text{s}^2$
Power	Scalar	+, 0, -	watt	W	$\text{kg m}^2/\text{s}^3$

50. (a) Work done on a particle undergoing a displacement from position A to position B .
- (b) The scalar summation of the work done by each force acting on a particle - superposition of work principle.
- (c) The resultant force which acts upon a particle. It is the same as $\sum_i \vec{F}_i$.
- (d) An infinitesimal displacement of a particle. It is a vector. Contrast it to \vec{s} , which is a finite displacement.
- (e) The kinetic energy of a particle. For a particle of mass m moving with a velocity v , the kinetic energy is $mv^2/2$.
- (f) An element of work done on a particle by a force during an infinitesimal displacement $d\vec{s}$.
- (g) The change in kinetic energy of a particle; $mv_f^2/2 - mv_i^2/2$, where v_f is the velocity at the final position (f) and v_i is the velocity at the initial position (i).
- (h) The average power, (W/t) exerted during a time interval t , where the total work done during this interval is W .
- (i) A force that varies in magnitude and direction as a function of position.
51. (a) Constant force, not necessarily parallel to finite displacement.
- (b) Constant force, parallel to finite displacement. Although $W = Fs$ may be the easiest form of the work definition to recall, remember that it is just a special case of a more general relationship.
- (c) Constant force, parallel to an infinitesimal displacement.
- (d) Constant force, not parallel to an infinitesimal displacement.
- (e) Variable force which depends on position.
52. (a) ok
- (b) NO - must have a ds under integral sign.
- (c) ok
- (d) No - left side implies increment, right side a finite amount.

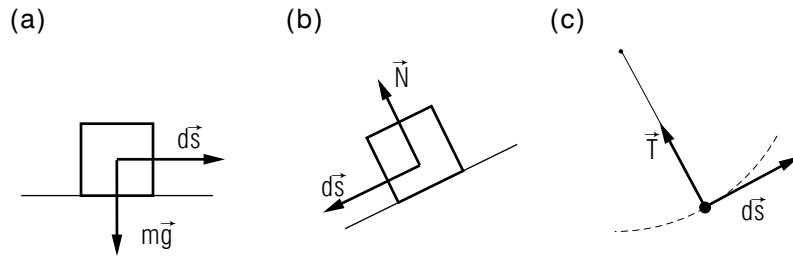
- (e) No - must have integral sign on right side.
- (f) No, must have vector multiplication sign (dot) between \vec{F} and $d\vec{s}$.
- (g) ok
- (h) ok
- (i) ok
- (j) No, just have vector multiplication sign (dot) between \vec{F} and \vec{v} .
53. Consider the work done by gravity on a round trip, when you throw a ball up into the air vertically and it returns to your hand.
- $$W = \int \vec{F} \cdot d\vec{s}$$
- $$W = W_{\text{up}} + W_{\text{down}} = \int mg(-1) ds + \int mg(+1) ds = 0 \text{ because net } \vec{s} = \int d\vec{s} = 0.$$
- For any round trip displacement net $\vec{s} = 0$.
54. $\vec{F} = 0$, $d\vec{s} = 0$ and $\vec{F} \perp d\vec{s}$.

If a constant force \vec{F} acts perpendicular to the displacement \vec{s} of a particle then $\theta = 90^\circ$ so that $W = 0$ and no work is done by this force. Note that work may not always be done on a particle which is undergoing a displacement even though a force is applied.



Here are three forces doing zero work (see figures below):

- (a) the force of gravity ($mg\hat{y}$) on a particle moving horizontally.
- (b) the normal force (\vec{N}) as the particle moves along the surface.
- (c) the tension (\vec{T}) in the cord of a simple pendulum, since this force is always \perp to the displacement.



55. Remember that $\hat{x} \cdot \hat{x} = 1$, $\hat{x} \cdot \hat{y} = \hat{x} \cdot \hat{z} = 0$, etc.
56. a) 3600 J
b) 7200 J
57. 3.6×10^3 J. Hint: $\vec{v} = d\vec{s}/dt$.
58. $\Delta E_k = 0$, since $\Delta E_k = m\Delta(v^2)/2 = m[(v + \Delta v)^2 - v^2]/2$, and $\Delta v = 0$.
59. $\vec{F}_R = 0$, that is there is no net force parallel to the displacement.
60. (a) The resultant force acts on the particle in the direction of the displacement, so particle increases in energy (kinetic).
(b) The resultant force acting on the particle is opposite to the direction of the displacement, so that there is a decrease in kinetic energy.
(c) An increase in the energy of the system.
(d) A decrease in the energy of the system.
61. $4E_k = mv^2/2$, since v^2 is always positive the kinetic energy is positive. You can never have less than zero kinetic energy. Zero kinetic energy corresponds to an object at rest. Since kinetic energy is a scalar it does not tell you anything about the direction of the velocity of a particle.
62. 19 J
63. 480 J
64. a) 412 W; b) 2000 W *Help: [S-38]*

65. It provides a method for calculating the change in energy of a particle when the force acting on it is not constant but is a function of the position of the particle.
66. Work can be found from the ΔE_k without having to know anything about the nature of the forces involved.

Sect. 6: Hints for the *Problem Supplement*.

S-1 (from PS-problem 1)

- (a) At constant magnitude of the frictional force will equal the magnitude applied force. Resolve the applied force in its vertical and horizontal components. Remember that $N = mg - F \sin \theta$.
(b) The coefficient of friction is the frictional force divided by the normal force, so that $\mu = F \cos \theta / (mg - F \sin \theta)$.

S-2 (from PS-problem 2)

Determine $\vec{F} \cdot d\vec{s}$, applying the fact that $\hat{x} \cdot \hat{x} = 1$ and $\hat{x} \cdot \hat{y} = \hat{x} \cdot \hat{z} = 0$.

S-3 (from PS-problem 3)

The kinetic energy is given, so use $mv^2/2 = E_k$ and solve to find that $v = (2E_k/m)^{1/2}$.

S-4 (from PS-problem 4)

Recall that $|\vec{v}|^2 = v^2 = v_x^2 + v_y^2$.

S-5 (from PS-problem 5)

- (a) Calculate the work done by the escalator against gravity by resolving the force it exerts into vertical and horizontal components. Only the vertical component will do any work. Why?
(b) Use the definition of power involving work and time.

S-6 (from PS-problem 6)

Use the definition of power involving force and velocity.

S-7 (from PS-problem 7)

Draw a one-body diagram for the car. Resolve the forces into their components parallel and perpendicular to the street. Here we write the street-to-horizontal angle as ϕ , the cable-to-street angle as θ , the mass of the car as m , the tension in the cable as T , the distance along the street as s , the coefficient of friction as μ , and the work done by the truck on the car as W . Then for the work done by the truck (via the cable) on the car-earth system we get:

$$W = sT \cos \theta = mgs \cos \theta (\sin \phi + \mu \cos \phi) / (\cos \theta + \mu \sin \theta).$$

If you still have trouble, see *Help: [S-24]*.

S-8 (from PS-problem 8)

Since the particle is in equilibrium the sum of the horizontal components of force are zero, as is the sum of the vertical components. You should be able to show for the horizontal components that: $F - T \sin \phi = 0$, and for the vertical components, $T \cos \phi - mg = 0$. Eliminate T between these two equations to obtain F in terms of ϕ . Since $\phi = s/\ell$, then $ds = \ell d\phi$. Now evaluate $\int \vec{F} \cdot d\vec{s}$, remembering that there is an angle between \vec{F} and $d\vec{s}$.

S-9 (from PS-problem 9)

Apply the definitions of $\vec{v} = d\vec{s}/dt$ and E_k .

S-10 (from PS-problem 10)

Show that the work done is equal to $\int (mg\hat{z}) \cdot (d\vec{s})$ where \vec{s} turns out to be the net vertical displacement.

S-11 (from PS-problem 11)

For your analytical solution write the equation for the straight line shown which will represent the effective force. This is $F = 10 - 2x$.

S-13 (from PS-problem 13)

Apply the work-kinetic energy relation, assuming that the retarding force is constant during the displacement, to find the distance the bumper moves from knowing the total work done using ΔE_k .

S-14 (from PS-problem 14)

Apply the work-kinetic energy relation for motion in a vertical direction: $v_f = [2gh + v_0^2]^{1/2}$. This method and the laws of linear motion should lead you to the same results.

S-15 (from PS-problem 15)

First apply the work-kinetic energy relation to find the average retarding force per unit thickness of metal plate, since the ΔE_k is known. Then apply the principle a second time with the new ΔE_k to find the distance required. Note that $v_f = 0$ for the second case.

S-16 (from PS-problem 16)

Use the work-kinetic energy relation, with $F_{\text{friction}} = \mu N = \mu mg$ so that $s = v_0^2 / (2\mu g)$.

S-17 (from PS-problem 17)

(a) Since $F = kx$, find k from the given values of F and x . Then integrate $\vec{F} \cdot d\vec{s}$ from 0 to 0.3 m to find the work done. Note that only 80% of this work is converted into E_k .

(b) $h = v_i^2 / (2g)$

(c) $F = [(mv_i^2/2) - (mgh')]/h'$ where $h' = 0.9h$.

S-18 (from PS-problem 18)

Here $x = y$ so that $dx = dy$. Use $\int F_x dx + \int F_y dy$ and simplify before integrating.

S-19 (from PS-problem 19)

Use $\int \vec{F} \cdot d\vec{s}$ with $\vec{F} = -kx^2\hat{x}$ and $d\vec{s} = dx\hat{x}$.

S-20 (from PS-problem 20)

Use the work-kinetic energy relation, including the drag energy in the W expression, so that the power will equal $[W_{\text{drag}} + (Mv_f^2/2)]/t$ where M is the total mass.

S-21 (from PS-problem 21)

In the expression $W = \int_i^f \vec{F} \cdot d\vec{s}$:

(1) What is $d\vec{s}$? Help: [S-26]

(2) What is $\vec{F} \cdot d\vec{s}$? Help: [S-32]

(3) Use a *given* property of \vec{F} to simplify the integral before integrating. Help: [S-28]

Part (a): The other forces besides \vec{F} acting on the subject are the force due to the earth's gravity, and the force exerted on the object by the surface of the incline. Draw your vector diagram of the forces and resolve them into their components parallel and perpendicular to the inclined surface.

(4) Repeat questions (1) and (2) above for the force due to gravity. Help: [S-27] and Help: [S-35]

(5) Repeat questions (1) and (2) above for the force exerted by the surface. Help: [S-29] and Help: [S-1]

(6) What are the resulting force components parallel to the displacement? Help: [S-36]

Part (b): Now your initial velocity vector is *down* the incline, but $d\vec{s}$ is still up the incline. Why? Help: [S-34]

S-22 (from PS-problem 22)

(a) You should find $r = (K/g)^{1/2}$.

(b) This is a variable force: it depends upon position \vec{s} . So:

$$W_{R_e \rightarrow r} = \int_{R_e}^r \vec{F} \cdot d\vec{s}$$

Why must $\vec{F} \cdot d\vec{s}$ be evaluated under the integral sign? Help: [S-33]

(c) Here $r \rightarrow \infty$ so that the work done becomes $K m/R_e$. Why? Help: [S-37]

S-23 (from PS-problem 23)

$$F = \frac{1.60 \times 10^2 \times 550 \text{ ft lb/s}}{(56 \text{ mi/hr})(\text{hr}/3600 \text{ s})(5280 \text{ ft/mi})}$$

S-24 (from [S-7])

The work done by the cable is done against car-earth frictional (contact) and gravitational (non-contact) forces. $W_{\text{cable}} = \vec{T} \cdot \vec{s} = Ts \cos \theta$ and $T = mg(\mu \cos \phi + \sin \phi)/(\mu \sin \theta + \cos \theta)$.

Only for those interested: The work done against the car-earth gravitational force is $mgs \sin \phi = 1.24 \times 10^6 \text{ J}$. The work done against the car-earth frictional force is $\mu Ns = 0.23 \times 10^6 \text{ J}$ where:

$$N = mg(\cos \theta \cos \phi - \sin \theta \sin \phi)/(\mu \sin \theta + \cos \theta).$$

S-26 (from [S-21])

An infinitesimal displacement vector whose magnitude is the element of path length along the incline and whose direction is up the incline parallel to its surface.

S-27 (from [S-21])

Same as [S-26].

S-28 (from [S-21])

$\int \vec{F} \cdot d\vec{s} = \vec{F} \cdot \int d\vec{s}$ because F is constant. $\int d\vec{s}$ is just a vector along the incline whose magnitude is the total distance between the starting point and the ending point.

S-29 (from [S-21])

Same as [S-26].

S-31 (from [S-21])

Zero (be sure you know why).

S-32 (from [S-21])

$(F \cos \theta)ds$ where ds is element of path length up the incline.

S-33 (from [S-22])

The unit vector \hat{r} is radially outward, by definition from mathematics. The force you exert is in the same direction so:

$$\vec{F} = \frac{K m}{r^2} (\hat{r}) .$$

The displacement was described as being in the direction away from the center of the earth so: $d\vec{s} = \hat{r} dr$. Recall that the scalar product of any unit vector with itself is unity so:

$$\vec{F} \cdot d\vec{s} = \frac{K m}{r^2} dr .$$

S-34 (from [S-21])

Since it is only the final displacement D which is of interest.

S-35 (from [S-21])

$-(Mg \sin \theta)ds$, where ds is element of path length up the incline.

S-36 (from [S-21])

$(F \cos \theta - mg \sin \theta)$.

S-37 (from [S-22])

As $r \rightarrow \infty$, $\frac{1}{r} \rightarrow 0$.

S-38 (from AS-problem (r))

velocity of block was said to be constant, so acceleration of block is zero, so net force on block is zero.

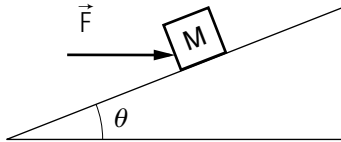
S-39 (from AS-Answer (43))

During this short time interval, the angle between the path of the football and the force of gravity is ...

MODEL EXAM

1. See Output Skills K1-K5 in this module's *ID Sheet*. One or more, or none, of these skills may be on the actual exam.

2.



A constant horizontal force \vec{F} pushes an object of mass M up a frictionless incline which makes an angle θ with the horizontal.

- Starting with the object midway *up* the incline, moving the speed of v_0 directed up along the incline, use the work-kinetic energy relation to find the velocity of the object when it's at a point a distance D further up the incline.
 - If at this same starting point as in part (a) the object started with the speed v_0 directed *down* along the incline (same forces acting as before), use the work-kinetic energy relation to find the velocity of the object when it's at the point a distance D further up the incline.
 - Explain the relation between your answers to parts (a) and (b). What is the difference in the overall motion between the two cases?
3. The gravitational force on an object of mass m which is at or above the surface of the earth, say at a total distance r from the center of the earth, has a magnitude $K m/r^2$ and is directed toward the center of the earth. Here K is a constant equal to $3.99 \times 10^{14} \text{ m}^3/\text{s}^2$. Hence the force is not constant but diminishes the farther the object is from the earth's center.
- At the surface of the earth, recall from previous knowledge that this force is mg . With this knowledge of the force at the surface of the earth and with K given above, determine the radius of the earth.
 - What is the minimum amount of work that you must do (energy you must expend) in order to move an object of mass m radially outward from the surface of the earth to a distance r from the center of the earth ($r > R_e$)? (HINT: What's the *minimum* force you must exert? That determines the *minimum* work).

- From this result, calculate how much energy it takes to move a person of mass 65 kilograms from the surface of the earth to an infinite distance away (disregard gravity forces other than from the earth).
4. Suppose a car engine is delivering $1.60 \times 10^2 \text{ hp}$ at a continuous rate, keeping the car at 25 m/s (56 mph). Determine the force the engine is overcoming.

Brief Answers:

- See this module's *Text*.
- See this module's *Problem Supplement*, problem 21.
- See this module's *Problem Supplement*, problem 22.
- See this module's *Problem Supplement*, problem 23.

