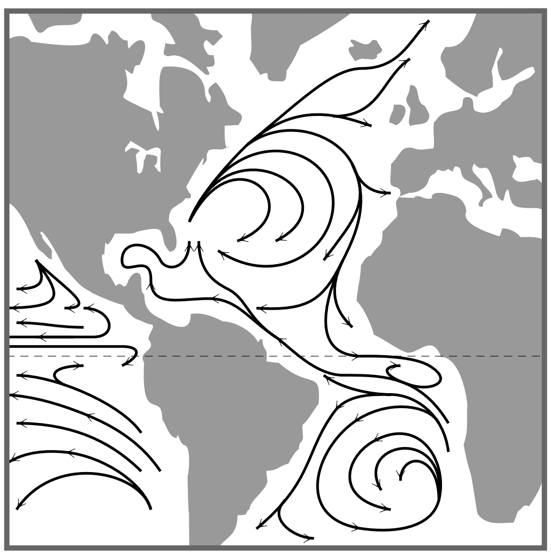
PHYSNET

MISN-0-18

ROTATING FRAMES OF REFERENCE; EFFECTS ON THE SURFACE OF THE EARTH



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

ROTATING FRAMES OF REFERENCE; EFFECTS ON THE SURFACE OF THE EARTH

by

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Title: **Rotating Frames of Reference; Effects on the Surface of the Earth**

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Input Skills:

1. Determine the direction and magnitude of the vector (cross) product of two given vectors (MISN-0-2).
2. State the vector relationships between velocity, angular velocity, position, and force for an object in uniform circular motion (MISN-0-17).
3. Transform velocities between different Galilean frames of reference and use double-subscript object-observer notation (MISN-0-11).

Output Skills (Knowledge):

K1. Derive the Coriolis and centrifugal accelerations, starting with the expression which relates a vector's time derivative in a rotating reference frame to the time derivative in a non-rotating reference frame.

K2. Derive the rotational directions of winds near high and low pressure centers in the northern and southern hemispheres.

K3. Explain the alternating prevailing east-west wind directions found as one moves in a north-south direction on the face of the earth.

K4. Explain (qualitatively) the reason for the apparent rotation of the plane of a Foucault pendulum, and explain how observations of it prove that the earth rotates.

K5. Demonstrate the Coriolis force using a rotating stool and show that, in all cases, the direction of the force is as given by the algebraic formula.

Post-Options:

1. "Relativistic Gravitation I; The Equivalence Principle" (MISN-0-110).

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OF PROJECT PHYSNET

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New authors, reviewers and field testers are welcome.

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1. Observing Time-Dependent Vectors

1a. Rotating, Non-Rotating Observers. We are going to compare observations of a vector $\vec{r}$'s change with time, as observed by a rotating (R) and a non-rotating (NR) observer. Imagine a large horizontal turntable rotating with angular velocity[1] $\vec{\omega}$, as shown in Fig. 1, with a radius vector $\vec{r}_o$ from the center of the turntable to an object (labeled "o") traveling with velocity $\vec{v}_o$. We will suppose that this velocity is being measured by an observer R who is stationary in the turntable's rotating frame, and also by an observer NR who is stationary in the non-rotating frame. What is the relation between the two velocity vectors of the object, the $\vec{v}_{\rm o,R}$ seen by the rotating observer and the $\vec{v}_{\rm o,NR}$ seen by the non-rotating observer?

1b. Observing Velocities. First, recall that the velocities of an object seen by two observers are related by:[2]

$$\vec{v}_{\rm o,A} = \vec{v}_{\rm o,B} = \vec{v}_{\rm B,A}\,,$$

[1]The vector angular velocity of a rigidly rotating object is defined by the direction that right-hand threads along the axis would move the object or, mathematically, by: $\vec{\omega} = \vec{v} \times \vec{r}/r^2$, where $\vec{v}$ and $\vec{r}$ are for any point on the rigidly-rotating object (for example, the turntable in Fig. 1).

[2]See "Relative Linear Motion, Frames of Reference" (MISN-0-11).

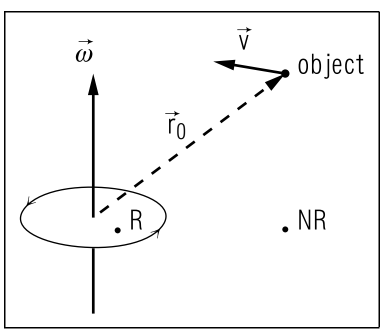

Figure 1. The symbol R labels a rotating frame, NR a non-rotating one (see text).

where $\vec{v}_{\rm o,A}$ is the velocity of the object as seen by observer A, $\vec{v}_{\rm o,B}$ is the velocity of the object as seen by observer B, and $\vec{v}_{\rm B,A}$ is the velocity of B as seen by A. For our case this becomes:

$$\vec{v}_{\rm o,NR} = \vec{v}_{\rm o,R} + \vec{v}_{\rm R,NR}\,.$$

The velocity of the rotating observer's frame, $\vec{v}_{\rm R,NR}$, must be measured at the position of the object and is therefore given by[3]

$$\vec{v}_{\rm R,NR} = \vec{\omega} \times \vec{r}_{\rm o}\,, \tag{1}$$

hence the velocities of the object measured by the two observers are related by:

$$\vec{v}_{\rm o,NR} = \vec{v}_{\rm o,R} + \vec{\omega} \times \vec{r}_{\rm o}\,. \tag{2}$$

1c. Observing Arbitrary Vectors. We can use Eq. (2), just derived, to find the differing time rates-of-change seen by the two observers for arbitrary vectors. To do this we first note that $\vec{v}$ is equal to $d\vec{r}_{\rm o}/dt$, and then make use of the theorem that any general relation valid for one vector is valid for all vectors.[4] Substituting $d\vec{r}_{\rm o}/dt$ into Eq. (2), we get:

$$\left(\frac{d\vec{r}_{\rm o}}{dt}\right)_{\rm NR} = \left(\frac{d\vec{r}_{\rm o}}{dt}\right)_{\rm R} + \vec{\omega} \times \vec{r}.$$

Then, making use of the general theorem quoted above, we can replace the radius vector to the object by an arbitrary vector which we will call $\vec{G}$:

$$\left(\frac{d\vec{G}_{\rm o}}{dt}\right)_{\rm NR} = \left(\frac{d\vec{G}_{\rm o}}{dt}\right)_{\rm R} + \vec{\omega} \times \vec{G}. \tag{3}$$

Any vector can be substituted for $\vec{G}$ in this equation.

2. Coriolis and Centrifugal Effects

2a. Acceleration. In order to modify Newton's Second Law for use in a rotating frame of reference, we will first determine the acceleration in that frame relative to its value in the non-rotating frame where Newton's

[3]See "Kinematics: Circular Motion" (MISN-0-9).

[4]See "Mathematical Skills: Addition, Subtraction, and Products of Vectors" (MISN-0-2).

laws are valid. That is easily accomplished by substituting the velocity of an object, as seen from the non-rotating frame, into Eq. (3):

$$\left(\frac{d\vec{v}_{\rm o,NR}}{dt}\right)_{\rm NR} = \left(\frac{d\vec{v}_{\rm o,NR}}{dt}\right)_{\rm R} + \vec{\omega} \times \vec{v}_{\rm o,NR}.$$

Then substitute for $\vec{v}_{\rm o,NR}$, Eq. (2), on the right side to obtain the relation between the observed accelerations:

$$\vec{a}_{\rm o,NR} = \vec{a}_{\rm o,R} + 2\vec{\omega} \times \vec{v}_{\rm o,R} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_o)\ .$$

We can now solve for the acceleration of the object as seen by the rotating observer:

$$\vec{a}_{\rm o,R} = \vec{a}_{\rm o,NR} - 2\vec{\omega} \times \vec{v}_{\rm o,R} - \vec{\omega} \times (\vec{\omega} \times \vec{r}_{\rm o})\,. \tag{4}$$

It is interesting to evaluate this equation for $\vec{a}_{\rm o,R}$ when $\vec{a}_{\rm o,NR}$ is zero, when $\vec{\omega}$ is zero, when $\vec{v}_{\rm o,R}$ is zero, when $\vec{v}_{\rm o,R}$ is parallel to $\vec{\omega}$, etc., in order to make sure that they agree with what one would expect.

The two "correction" terms in Eq. (4) are called the Coriolis and Centrifugal terms, respectively. Note that they both disappear as the angular velocity of the rotating observer goes to zero. If the object is motionless in the rotating frame then the Coriolis term goes away. If the object is crossing the axis of rotation the centrifugal term is zero.

2b. The Forces. The apparent forces acting on an object, as seen from a rotating frame, can be determined by simply multiplying Eq. (4) by the object's mass m_o and applying Newton's Second Law in the non-rotating frame where it is valid:

$$\vec{F}_{\rm o,R} = \vec{F}_{true} + \vec{F}_{\rm Coriolis} + \vec{F}_{\rm centrifugal}\,, \tag{5}$$

where

$$\vec{F}_{\rm Coriolis} = -2m\vec{\omega} \times \vec{v}_{\rm o,R}\,, \tag{6}$$

$$\vec{F}_{\rm centrifugal} = -m\vec{\omega} \times (\vec{\omega} \times \vec{r}_{\rm o})\,. \tag{7}$$

The latter two forces are fictitious: they only seem to be there because one is observing from a rotating frame and thinking of it as stationary.

The fictitious Coriolis and centrifugal forces are often perceived by an observer in a rotating frame as forces which must be "canceled out" by real restraining forces in order to make an object follow a desired path. In reality it is only the restraining forces that are real: they produce the required non-rotating-frame accelerations necessary to keep the object on its prescribed path.[5]

[5]See "Relativistic Gravitation I: The Equivalence Principle" (MISN-0-110) for a discussion of the equivalence of gravity and acceleration; necessary for attempts to answer the question: How does one explain the observed Coriolis force if one believes that one can equally well describe the earth as non-rotating and the rest of the universe as rotating about it?

3. The Foucault Pendulum

3a. Introduction. One almost always finds a Foucault[6] pendulum hanging in a science museum, and frequently the pendulum is several stories high. The principal aim of showing one remains as it was with Foucault; namely, to demonstrate that the earth rotates on its axis with a period of one day. This is meant to be in contrast to a picture in which the earth is taken as non-rotating and the sun is taken as following a daily circular traversal of the earth.

The suspension system of a Foucault pendulum is constructed in such a way as to allow it to swing as freely as possible. As a first approximation, we can consider it to be allowed to swing freely in any plane, even a rotating one.

3b. A Function of Latitude. Think of transporting the pendulum to the North pole of the Earth, where it is suspended vertically, as usual, and set to swinging back and forth over the pole. As seen from the inertial (non-rotating) frame of reference, the pendulum's swings lie in a non-rotating plane. As seen by an observer at rest on the rotating earth, the plane of swinging would appear to rotate with an angular velocity which is just the negative of the angular velocity of the earth:

$$\vec{\omega}_p = -\vec{\omega}_e\,,$$

where $\vec{\omega}_p$ is called the (apparent) precessional angular velocity of the pendulum.

Now think of moving the same pendulum to the equator. There it would be forced to go around with the earth while swinging, and the precession would be absent.

It is not hard to convince oneself that the precessional rate is determined by the component of the earth's angular velocity along the pendulum's "vertical," and earth radial:

$$\omega_p = \omega_e \sin\theta_e\,, \tag{8}$$

where θ_e is the latitude of the suspension point (0° at the equator, 90° at the north pole). Equation (8) can be derived exactly from the requirements:

[6]pronounced "foo cō*l*" in English.

(i) $\omega_p = \omega_e$ at the north pole;

(ii) $\omega_p = 0$ at the equator;

(iii) the Coriolis force, which is causing the precession, depends on the sine of the angle between $\vec{\omega}$ and $\vec{v}$, and this can be directly shown to be equal to the angle of latitude, θ_e.

It is quite apparent that the precessional rate of a Foucault pendulum can be combined with the latitude angle of its position to give the absolute rotational rate of the earth.

4. Air Currents near Pressure Centers

The spiral appearance of cyclonic cloud formations seen in weather satellite photographs is due to the Coriolis force: it constitutes dramatic evidence of the rotation of the earth. The common spiral satellite photographs shown in texts and on TV weather reports are of low-pressure centers. The air currents in the vicinity of such a center move over the face of the earth toward it and then rise from the surface and thereby produce the lowered pressure. From traveling over the surface these currents are warm and moisture-laden, bringing storms and being made visible by their clouds. These routinely-seen patterns are called extratropical cyclones. on the other hand, high pressure centers are produced by air descending from the upper atmosphere and then spreading out away from the center. Such air is typically cold and clear and does not show up in satellite photographs.

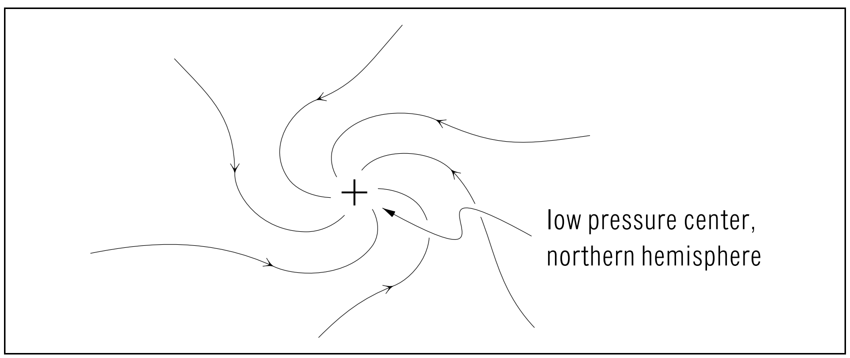

Figure 2. Appearance of the air currents near a low pressure center in the northern hemisphere, as seen from a weather satellite.

As air moves toward or away from a pressure center, the Coriolis force produces a sideways deflection: the result is a spiral trajectory as in Fig. 2. It is obviously quite easy to obtain the direction of rotation of these spirals in each hemisphere, for each type of pressure.

5. Prevailing Wind Directions

As one proceeds along a meridian[7] on the face of the earth, the prevailing wind directions alternate. This fact was all-important in the days of square-rigged clipper ships that could only sail in the direction the winds were blowing. For example, they sailed the Atlantic from America to Europe following the prevailing west wind along 30° north latitude and then returned using the prevailing east wind along 45° north latitude. These prevailing-wind patterns are due to the Coriolis force, as is the direction of the prevailing wind in our vicinity.

The alternation of prevailing wind directions is due to a combination of differential air heating and the Coriolis force. Begin by imagining an air mass covering a portion of the earth's surface in the northern hemisphere. The part nearer the equator is heated more than the rest and therefore tends to rise. It is easy to see that the Coriolis force deflects it to the west, resulting in rising "east winds." Continuing motion results in a deflection northward and then the cooled air moves downward and is deflected easterly, resulting in descending "west winds." The whole constitutes a large air cell. The direction of circulation of such air cells in the southern hemisphere can be easily determined and these patterns confirm the rotation of the earth.

Acknowledgments

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[7] A meridian is a north-south line on a map.

LOCAL GUIDE

A DEMONSTRATION

You are to predict the direction of a particular Coriolis force acting on the CBI Consultant, explaining in detail (to the Consultant) how you manipulated the cross products in order to make the prediction. You are then to do a demonstration on the Consultant to prove that you are correct.

Practice this demonstration on yourself or a friend before trying it on the Consultant: you get only one chance!

Demonstration: You will have the Consultant note the sideways deflection of his or her **fist** while pulling it inward or outward during rotation. You are to explain the direction of the deflection to the Consultant, pointing out the direction of each vector in the Coriolis force and how the "cross products" produce the observed deflection.

You will ask the Consultant to sit in the rotating chair in the corner of the CBI Consulting Room. You will ask the Consultant to keep his or her head on the axis of rotation at all times.

Then you will describe what is going to take place and you will tell the Consultant the directions of the various vectors involved and what direction that predicts for the Coriolis force.

You will ask the Consultant to fully extend his or her arm and you will then rotate the chair slowly in the direction you specified in advance to the Consultant. Have the Consultant do several of: arm in, arm out, CW rotation, CCW rotation. In each case you predict the direction of the Coriolis force in advance.

When finished, have the Consultant give you a signed receipt describing how well you did, on a scale of 0-4, which you can attach to the written part of the exam.

READINGS

(See next page for readings-skills cross-reference)

MOD: This module.

AF: M. Alonso and E. Finn, *Physics*, Addison-Wesley (1970). Section 6.5 includes a beautiful Tiros satellite photograph showing wind-driven clouds spiraling in toward a low-pressure center, plus Foucault pendulum diagrams.

HRI: D. Halliday and R. Resnick, *Physics*, Part I, Wiley (1977). Section 6-4 includes a discussion of "Forces and Pseudo-Forces," not including the Coriolis force.

BO: V. Barger and M. Olsson, *Classical Mechanics*, McGraw-Hill Book Co., N.Y. (1973). Sections 6-1 to 6-4 include a picture and derivations for the Foucault pendulum and a short discussion of trade winds and cyclonic effects.

AM: J. Ames and F. Murnahan, *Theoretical Mechanics*, Dover Publ., Inc., N.Y. (1957). Section 67 includes the full theory of the Foucault pendulum.

KKR: C. Kittel, *Mechanics (Berkeley Physics Course - Vol. 1)*, W. Knight, and M. Ruderman, McGraw-Hill Book Co., N.Y. (1965). Pages 84-91 contain a useful discussion of the definition of vector angular velocity. The "Coriolis acceleration" is given with the opposite sign to that generally used. The derivation does not use vector notation and it is noted that this method produces "some tedious algebra."

JM: J. Marion, *Classical Dynamics*, Academic Press, N.Y. (1970). Chapter 11 has a footnote at the bottom of page 346 that is priceless and well worth a trip to the library. Figure 12-4, showing cyclonic deflection, is quite clear.

READINGS-SKILLS CROSS REFERENCE

Skill	Ref.	Items
K1 (derive forces)	MOD	Sect. 2
	JM	Ch. 11
K2 (cyclonic rot.)	MOD	Sect. 4
	JM	Ch. 11
	BO	Ch. 6
K3 (prevail. winds)	MOD	Sect. 5
	BO	Ch. 6
K4 (Foucault pend.)	MOD	Sect. 3
	AF	
	BO	Ch. 6
K5 (Demo., Cor. f.)	MOD	ME

MODEL EXAM

1-4. See Output Skills K1-K4 in this module's *ID Sheet*.

5. (Must be done in advance of taking the written part of the exam.) Predict the Coriolis force for the case you will demonstrate and then do the demonstration on the Consultant. Have the Consultant do several of: arm in, arm out, CW rotation, CCW rotation. In each case you predict the direction of the Coriolis force in advance.

   When finished, have the Consultant give you a signed receipt describing how well you did, on a scale of 0-4, which you will attach to the written part of the exam.