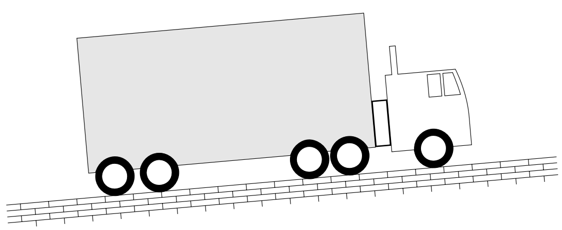
MISN-0-16

FRICTION IN APPLICATIONS OF NEWTON'S SECOND LAW



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

FRICTION IN APPLICATIONS OF NEWTON'S SECOND LAW

by
Peter Signell and Harry Dulaney

Title: **Friction in Applications of Newton's Second Law**

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Version: 5/17/2002 Evaluation: Stage 0

Length: 1 hr; 28 pages

Input Skills:

1. Solve problems with contact and non-contact forces, connected bodies, and constant forces, without friction (MISN-0-10).

Output Skills (Knowledge):

K1. Vocabulary: coefficient of non-sliding friction, coefficient of sliding friction, force of non-sliding friction, force of sliding friction.

Output Skills (Rule Application):

R1. Interpret the starting and stopping of motion of given objects such as walking persons, cars, bicycles, boats, and planes in terms of Newton's three laws and forces of non-sliding and sliding friction.

Output Skills (Problem Solving):

S1. Given sufficient information on one or more masses, possibly connected, possibly on inclines, possibly with zero or non-zero non-sliding and/or sliding coefficients of friction, and possibly with constant applied forces, solve for requested values of forces, masses, coefficients of friction, and kinematic variables.

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1. Introduction

1a. Friction Needs Normal and Tangential Forces. In this module we introduce some effects of friction into some simple cases in mechanics.[1] All such cases occur when there are two objects in contact and there are two force components that act on the objects: (1) a "normal force" which is perpendicular to the surfaces and whose fuction is to keep the surfaces in contact; and (2) a "tangential or lateral force" which is parallel to the surfaces and which would cause one surface to slide over the other if there was no friction. These two forces are at right angles to each other: one is normal to the two surfaces that are in contact and the other is tangential to those surfaces (see Fig. 1). The normal force acting on one of the objects in contact is generally equal to, and in the opposite direction from, the normal force acting on the other object in contact with the first: in this way contact is maintained between the surfaces of the two objects and there is no motion in the direction perpendicular to the surfaces (vertically in Fig. 1). Similarly, the tangential or lateral force on one object is generally in the opposite direction to the lateral force (horizontally in Fig. 1) acting on the other object and this means that one object would slide over the other if there was no friction.

1b. Non-Sliding and Sliding Friction. There are many different kinds of friction, but the major ones are non-sliding fricxtion and sliding friction. An example of *sliding* friction is the force between the wheel of a stuck car and the icy surface that the wheel is spinning on, a force that may cause smoke to be generated but no motion of the car. Another

[1] See "Frictionless Applications ..." (MISN-0-10).

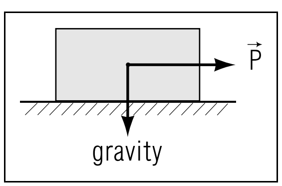


Figure 1. Some forces acting on a pulled block. Reactive forces acting on the block are not shown.

example is the car sliding across an icy patch on a road, where the brakes have stopped the tires from turning but the tires, and hence the car, are still moving across the ice. An example of *non-sliding* friction is the force between the tires of the same car and a bare part of the pavement or between the tires and the icy surface after sand has been applied to the ice so the car can "get a grip" and cause the car to be controllable again. Two sub-categories of sliding friction are *rolling* friction and *lubricated-surface* friction, but we will not deal with these specifically in this module.

1c. Example: Hand Rubbing. Think of pressing your hands tightly together, exerting a strong "normal" force (so named because it is mathematically "normal" to the plane of the surfaces that are in contact). If you are exerting enough normal force, then exerting a small tangential force will not cause motion; one hand will not start rubbing over the other. This is because the force of non-sliding friction is resisting the relative motion. In fact, the force of non-sliding friction is exactly equal but opposite to your applied tangential force, resulting in zero net force on your hand and hence zero acceleration of the hand.

If you now exert enough tangential force, it will "break" the non-sliding friction and allow relative motion (called "rubbing" in the case of hands). If the normal force is strong enough and the rubbing is fast enough and lasts long enough, you will perceive heat at the interface between the hands.

If you make the normal force smaller by pressing your hands together less tightly, you will find that a lower normal force results in a lower tangential force needed to break the force of non-sliding friction and allow motion.

2. Origin and Laws

2a. The Molecular Origin of Friction. When normal forces hold two surfaces together and tangential forces tend to make one surface slide over the other, the tangential force is opposed by a "frictional" force. This frictional force is due to chains of molecules that have settled out of the environment (dust, fingerprints, water vapor, etc.) onto each of the surfaces before they were in contact. Each clean surface had regular hills and valleys on the molecular level before the "dirt" arrived but the molecular chains of dirt are somewat irregular and can be caused to move on the surfaces. When the two surfaces are brought together, the dirt chains "lock" the surfaces together until enough sideways force is applied to cause a slipping from one surface valley to the next.[2] Thus if enough lateral force is applied there will be a sliding of one surface over the other but, if the force is then removed, the motion will slow down and stop. If an applied force is insufficient to overcome the "locking" effect of the dirt, the surfaces will stay "stuck" and one surface will not move over the other. The fact that there is no lateral motion in this locked case means that the lateral speed stays zero so the lateral acceleration stays zero so the net lateral force on each object is zero. Since we are applying a lateral force to each object, the fact that the net force on each object is zero means that the surface friction must be resisting motion with an exactly equal and opposite lateral resisting force on each onject. If we apply a zero lateral force and gradually increase it, friction will produce an exactly equal and opposite force that prevents any acceleration and hence any motion. However, when a large enough lateral force is applied, the "locking" of the surfaces is overcome and the surfaces start to slide over each other in a motion that appears smooth on the macroscopic level but that is a sort of "up and down", "slip and lock" motion on the molecular level.

2b. Amontons's Laws. This is the explanation of the widely-used Amontons's Laws for the force of friction between two surfaces[3] that are in contact.[4] which state that the (lateral) force of static friction between the two surfaces is: (1) proportional to the normal force holding the objects' surfaces together; and (2) independent of the surface area in contact.

2c. "Static" and "Kinetic" Friction. In technical documents, the friction in the case where a car is static, not moving because the wheel is slipping, is called "kinetic" friction, while the friction in the case where a car is moving without slipping, and hence has kinetic energy, is called the "static" case. To make clear which case is appropriate in particular circumstances, we will here use the terms "non-sliding" and "sliding" rather than "static" and "kinetic." However, you must usually use "static" and "kinetic" when finding friction characteristics in the literature.

[2]See "Surface Grime Explains Friction," *Physical Review Focus*, `http://focus.aps.org/v7/st6.html`, and references contained therein. There is a later preprint dated 4/30/02: *Statistical Mechanics of Static and Low-velocity Kinetic Friction*, by Martin H. Müser, Michael Urbakh, and Mark O. Robbins, Dept. of Physics and Astronomy, The Johns Hopkins University.

[3]The description of a force as being "between two surfaces" means that each of the objects whose surfaces are in contact experiences a force having the same physical origin. The two forces have equal magnitude but are opposite in direction.

[4]Guillaume Amontons, *Histoire de l'Académie Royale des Sciences avec les Mémoires de Mathématique ey de Physique*, page 206 (1699).

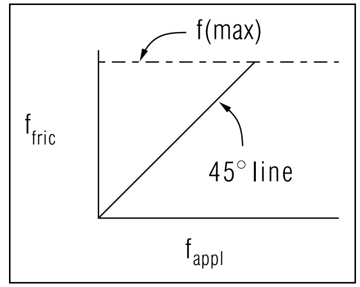


Figure 2. Response of a non-sliding frictional force to an applied force.

2d. General Characteristics. The force of non-sliding friction is tangential to two surfaces that are in contact and it increases as necessary in order to prevent relative motion of the two surfaces (see Fig. 2). It can increase in this fashion only up to some maximum that is determined: (1) by the nature of the two surfaces; and (2) by the magnitude of the normal force that is pressing the two surfaces together.

Note that the force of non-sliding friction itself depends only on the applied tangential force: it is only its maximum value which depends on the surface character and on the size of the normal force.

If an applied tangential force exceeds the capacity of non-sliding friction to resist it, then one surface starts to move over the other and the force of friction becomes a "sliding" one. The magnitude of this sliding force depends on: (1) the nature of the surfaces; (2) the magnitude of the normal force pressing the surfaces together; and (3) the relative speed of the surfaces (the speed of either surface with respect to the other).

Sliding friction decreases with increasing speed, while non-sliding friction decreases with decreasing applied tangential force.

2e. Example: A Block Pulled Horizontally. An object rests on a horizontal surface and a force $\vec{P}$ is applied to the block parallel to the surface (Fig. 3). From experience we know that if the force $\vec{P}$ is not too large the block will not move. Therefore the surface must be exerting a tangential force on the block that is equal but opposite to $\vec{P}$.

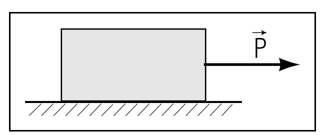


Figure 3. A horizontal force is applied to a block.

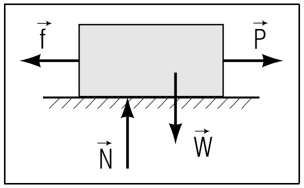


Figure 4. The force diagram for Fig. 3.

In the one-body diagram, Fig. 4, the force of the surface on the block has been decomposed into its "frictional" and "normal" components, labeled $\vec{r}_{ns}$. If the pull, $\vec{P}$, is increased, $\vec{f}$ will increase as long as its limit is not exceeded. If $\vec{P}$ is reversed (applied in the opposite direction) $\vec{f}$ will respond by reversing its direction. In a similar fashion, $\vec{N}$ is exactly equal but opposite to the non-contact gravitational pull of the earth on the object, $m\vec{g}$. Thus both the normal and frictional components of the force of the surface on the block act so as to maintain the block's state of equilibrium (keeping the resultant force zero: $F_{R,x} = 0, F_{R,y} = 0$).

When $|\vec{P}|$ exceeds the maximum sustainable force of non-sliding friction for the current value of the normal force, the block accelerates to the right. Now the force of sliding friction takes over and it increases slightly as speed increases. If $\vec{P}$ is removed completely, while the block is in motion, the force of sliding friction will continue to resist the motion until it brings the block to rest.

3. The Coefficients of Friction

3a. The Non-Sliding Case. It has been found experimentally that the maximum (tangential) force of non-sliding friction, $\vec{f}_{ns}(\text{max})$, which any particular surface can exert on another particular surface, is directly proportional to the normal force $\vec{N}$ that is pressing the two surfaces together:

$$f_{ns}(\text{max}) = \mu_{ns}\, N\,.$$

The constant of proportionality, μ_{ns}, is called the "coefficient of non-sliding friction."[5] The coefficient of non-sliding friction, μ_{ns}, depends on the nature of the two surfaces involved. It is a number without units (because it is a ratio of forces). It is very large for rough rubber on rough rubber and very small for ice skates on ice.

[5]In scientific documents it is called the "coefficient of static friction" and is labeled $\mu_{\rm s}$.

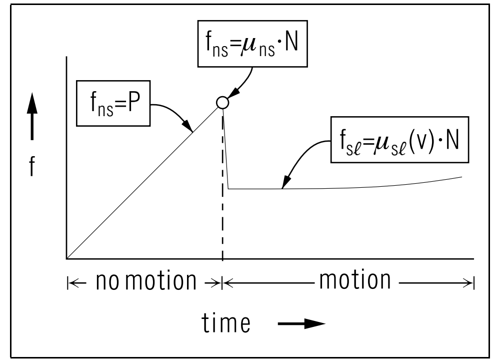


Figure 5. Frictional response as P of Fig. 3 increases linearly with time.

3b. The Sliding Case. If one surface is moving at speed v with respect to another surface, to which it is being pressed with normal force N, then the tangential force of friction is found to be proportional to N:

$$f_{sl} = \mu_{sl}\, N\,.$$

where μ_{sl} is called the "coefficient of sliding friction"[6] Experimentally, the coefficient of sliding friction is generally a very slowly increasing function of velocity; we shall assume that it is a constant, independent of velocity (for further details, the Appendix).

3c. The Transition. The transition from the non-sliding case to the sliding case is continuous, as are all processes in nature. To see this we plot, in Fig. 5, the frictional force versus time as the applied force P is increased at a constant rate. Note that on the left side of the graph the frictional force is of the non-sliding type, while on the right side it is of the sliding type. The abrupt change in the transition has been likened to a phase change in the material.[2]

3d. The Speed-Independent Approximation. Although the coefficient of sliding friction is a slowly-varying function of the relative speed of the two surfaces, it makes life easier and it is usually a good approximation to represent μ_{sl} by a single value, independent of relative speed:

$$f_{sl} = \mu_{sl}\, N\,. \qquad \text{(constant)}\,.$$

[6]in scientific documents it is called the "coefficient of kinetic friction" and is labeled $\mu_{\rm k}$.

This makes the sliding force and the maximum non-sliding force seem somewhat similar:

$$f_{ns}(\text{max}) = \mu_{ns}\, N\,,$$
$$f_{sl} = \mu_{sl}\, N\,.$$

4. A Numerical Example

4a. Applied Force $\boldsymbol{P}$ on Block on Inclined Plane. As an example of friction consider a 10 lb block placed on a 37° incline. A force $\vec{P}$ is applied to the block as shown in Fig. 6. For the block and surface assume: $\mu_{ns} = 0.30$, $\mu_{sl} = 0.20$. The block is initially at rest and $\vec{P}$ is 5.0 lb. The value of $\vec{P}$ is then increased in steps. Determine the frictional force acting on the block for these values of P: (a) 5.0 lb, (b) 6.0 lb, (c) 8.0 lb, (d) 10.0 lb, (e) a subsequent drop to 7.6 lb.

The one-body diagram is also shown in Fig. 6 where the force of the surface on the block has been decomposed into its frictional and normal components and the other forces have been resolved into components parallel and perpendicular to the incline.

For all cases, $F_{R,y} = ma_y = 0$:

$$N - W\cos 37° = 0\,; \qquad N = 10\,\text{lb}\,(0.8) = 8\,\text{lb}\,.$$

Then:

$$f_{ns}(\text{max}) = \mu_{ns}\, N = 2.4\,\text{lb}\,.$$

4b. For $\boldsymbol{P = 5}$ lb. f_{ns} will assume a value such that $F_{R,x} = 0$. Then:

$$P - W\sin 37° - f_{ns} = 0\,, \qquad f_{ns} = 5.0\,\text{lb} - (10\,\text{lb} \times 0.60) = -1.0\,\text{lb}\,.$$

Since 1.0 lb is less than $f_{ns}(\text{max})$, the frictional force is 1.0 lb and directed up the incline (as indicated by the negative sign). That is, $\vec{P}$ and $\vec{f}_{ns}$ together balance the component of weight that is along the incline.

4c. For $\boldsymbol{P = 6.0}$ lb. $F_{R,x} = 0$. Then:

$$P - W\sin 37° - f_{ns} = 0\,; \qquad f_{ns} = 6.0\,\text{lb} - 6.0\,\text{lb} = 0\,.$$

No frictional force is needed to maintain equilibrium so the frictional force is zero! The gravitational force component along the incline is just matched by the applied force $\vec{P}$.

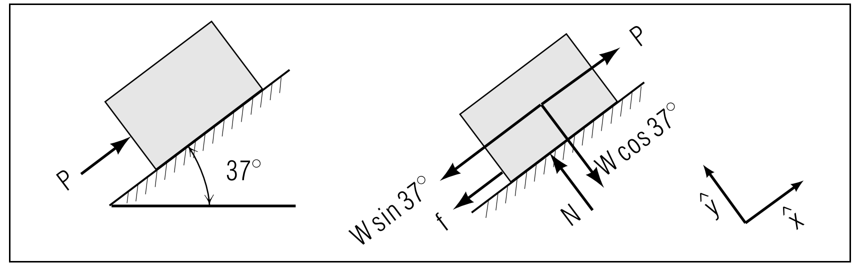


Figure 6. A block on an inclined plane and its one-body force diagram.

4d. For $\boldsymbol{P = 8.0}$ lb. $F_{R,x} = 0$. Then:

$$P - W\sin 37° - f_{ns} = 0\,; \qquad f_{ns} = 8.0\,\text{lb} - 6.0\,\text{lb} = 2.0\,\text{lb}\,,$$

and $\vec{f}_{ns}$ is directed along the incline downward. Now $\vec{P}$ has become so large that $\vec{f}_{ns}$ must be in the opposite direction so as to prevent motion.

4e. For $\boldsymbol{P = 10.0}$ lb.

$$P - W\sin 37° - f_{ns} = 0; \quad f_{ns} = 10.0\,\text{lb} - 6.0\,\text{lb} = 4.0\,\text{lb}\,,$$

and $\vec{f}_{ns}$ is directed down the incline downward. However, since 4.0 lb exceeds $f_{ns}(\text{max})$, f_{ns} is unable to assume the value 4.0 lb and $\vec{P}$ is too much for gravity plus non-sliding friction. There is a net force and the block will start to accelerate up the incline. Since the block is now sliding, the force of friction is that of sliding friction:

$$f_{sl} = \mu_{sl} N = (0.20)\,(8\,\text{lb}) = 1.6\,\text{lb}\,.$$

The net force resisting the acceleration is now 7.6 lb, so the resultant force on the block is 2.4 lb.

4f. A Subsequent Drop to $\boldsymbol{P = 7.6}$ lb. As a result of the 10.0 lb applied force, the block is accelerating up the incline. Now the value of P is dropped to 7.6 lb as stated in the problem. That value just balances the force of sliding friction and the component of weight down the incline, so the resultant force along the incline drops to zero. That means the acceleration drops to zero and the block subsequently moves up the incline with constant velocity (which is zero acceleration).

5. People, Animals, Vehicles

5a. Positive Acceleration. If you are standing still, you must start to accelerate in order to get up to normal walking speed. Your acceleration is in your direction of motion, the direction of increasing velocity, so by Newton's second law another object must be exerting a force on you in that direction. Your acceleration is equal to the net forward force, exerted on you by other objects, divided by your mass.

The earth is what exerts the "forward" force on you, the force in the direction in which you start moving. You initiate the process by exerting a *backward* force on the earth. In accordance with Newton's third law, the earth exerts an equal but oppositely directed *forward* force on you and that is the forward force which accelerates you!

The same holds true for cars, animals, and bicycles (along with their riders). Take a car, for example: the engine and drive-train components cause the tires to produce a backward force against the earth. The earth exerts the equal-but-opposite reactive force on the car. Of course that is the force *on the car* that accelerates the car in accordance with Newton's second law. The accelerating force is that of non-sliding friction since there is generally little slipping. Sliding friction comes into play only if, say, the operator of the vehicle applies so much force to the vehicle-earth interface that the wheels begin to slip and spin in place.

5b. Negative and Zero Acceleration. Car and bicycle brakes cause deceleration through the frictional force generated by one surface in the brake sliding over another with which it is held in tight contact. Thus the force causing deceleration while braking in such vehicles is that of purely sliding friction. Note, however, that if a vehicle is being kept stationary (not moving at all) by brake force, then that force is a non-sliding one.

5c. Jets and Rockets. No external force accelerates a jet or rocket operating in a vacuum. The air in the upper atmosphere is sufficiently thin so as to make this a reasonable approximation for jet planes operating there. With zero external force, Newton's second law says that the craft (together with its contents) must have zero acceleration and so it cannot speed up or slow down. However, we know from experience that such craft do speed up and slow down while in the upper atmosphere. The apparent discrepancy disappears if we watch the entire contents of the plane, not just the passengers and the plane's outer shell. During acceleration the fuel inside the plane is being burned and is thereby caused to exit rearward at an extremely high speed. Thus part of the "craft plus

contents" moves to a higher speed forward but another part of it acquires a high speed in the opposite direction. This is similar to the way one can throw beanbags rearward in order to gain forward speed while on ice skates or on an air sled. In cases like the jet plane, where there is no external force on the total object so its total mass must travel at constant speed, a special construct called the object's "center of mass"[7] is helpful for the discussion. For the present purpose, consider an object's center-of-mass to be a mathematical point located at the "average" position of the object's mass components (which is usually not very far from the object's geometrical center). The point here is that the center of mass of the totality of parts travels at constant speed regardless of accelerations and decelerations of individual parts. In our present case, the center of mass of the plane-plus-fuel moves along at a constant speed regardless of accelerations and decelerations of the plane alone. When the plane moves to a higher speed the center of mass remains at the old speed because fuel mass was shot rearward in the process of acceleration. The force that produces the plane's acceleration is exerted on the plane by the burning fuel in the firing chambers of the plane's engines. This force is exerted only on the forward parts of the chambers because the backward parts are open to the outside vacuum. For a deceleration, the jet of burned fuel must be sent out in the forward direction.

Acknowledgments

Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

A. Velocity Dependence of Sliding Friction

Only for Those Interested. For a wide variety of surface combinations, the velocity dependence of the coefficient of sliding friction is a very slowly rising logarithmic function of velocity:

$$\mu_{sl}(v) = \mu_{sl}(v_0) + a\,\ell n\,(v/v_0) \qquad (\text{for } v \geq v_0)\,,$$

[7] For a rigorous discussion of the "center of mass" concept, see "Tools for Static Equilibrium," MISN-0-5, or "Derivation of the Constants of the Motion for Central Forces," MISN-0-58. For a discussion of rocket propulsion, see "Mass Changing with Time: The Vertical Rocket, etc.," MISN-0-19

where v_0, $\mu_{sl}(v_0)$, and a are constants that depend only on the nature of the surfaces. If there is sliding friction at a particular speed and then the speed is increased or decreased, there is a very strong but short-lived upward or downward spike in the coefficient of sliding friction, followed by the new value of $\mu_{sl}(v)$. There are interesting graphs showing this effect in the preprint: *Statistical Mechanics of Static and Low-velocity Kinetic Friction*, by Martin H. Müser, Michael Urbakh, and Mark O. Robbins, Dept. of Physics and Astronomy, The Johns Hopkins University (4/30/2002).

PROBLEM SUPPLEMENT

ADVICE: Treat each problem as having five separate parts:

a. Draw a one-body diagram showing all relevant forces acting on the relevant object, with symbols representing all quantities;

b. Draw a diagram similar to the one in part (a), but with the forces now represented only by their components, where one axis is along the direction of acceleration and the axes are labeled;

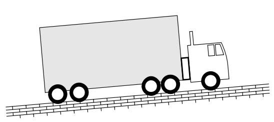
c. Write Newton's second law equations for the force components; and

d. Solve for the target unknown(s), still with all quantities represented by symbols.

e. Substitute numerical values along with their units and solve for the numerical answer(s). Express vector answers in terms of the unit vectors for the axes shown in part (b).

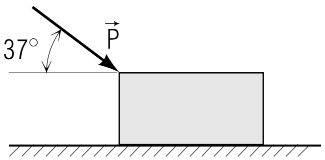
ADVICE: In step (a) above, suppose the direction of some force is unknown. For example consider the first problem below. There you must at least partially solve the problem before you know whether the roadbed exerts a force on the truck that is "up" the incline or "down" it. That force is unknown in the beginning so at that point you assign it a symbol and a direction (see the figure in Brief Answer 1a). If you pick the wrong direction, all that will happen is that you will get a negative value when it comes out in the solution.

1. A 16 ton truck has a deceleration of 1.0 ft/s^2 as it moves up a 5.0% grade (a 5% grade means that for every 100 ft traveled, you rise 5 ft vertically). Determine the force $\vec{p}$ of the road on the truck (see ADVICE above).



2. A farmer is taking a loaded wagon of produce to market (weight of wagon plus produce is 1800 lb). The wagon is pulled by a horse weighing 810 lb. The farmer wishes to uniformly increase the speed of the wagon by 22 ft/s in 5.0 s. A frictional force of 290 lb acts on the wagon. Determine the force with which the horse must push backward on the ground, and determine the tension the harness (connecting the horse to the wagon) must be able to withstand.

3. A 2.0 kg block is at rest on a horizontal surface. A force $\vec{P}$ is then applied to the block as shown in the figure. The coefficient of non-sliding friction between the block and surface is 0.30. If $P = 4.0\,\mathrm{N}$ is applied to

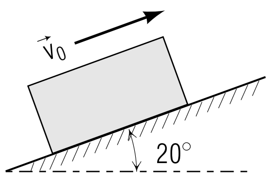


the block at rest, what frictional force will act on the block?

4. How large a force P must be applied to the block in Problem 3 in order to make it move?

5. Once the block in Problem 4 starts to move, it is observed that a force P of 12 N causes the block to have an acceleration of $2.0\,\mathrm{m/s^2}$. Determine the coefficient of sliding friction.

6. A block weighing 8.0 lb is moving up an incline of 20.0°. At some instant it has velocity $\vec{v_0}$ as shown in the sketch. For the interface between the block and the surface of the incline, $\mu_{ns} = 0.40$ and $\mu_{sl} = 0.30$. Only gravity and the surface of the incline exert forces on the block. Determine the acceleration of the block.



7. When the block in Problem 6 stops, will it remain at rest or will it slide back down the incline? If it remains at rest, what frictional force will be acting on it?

8. If the block in Problem 6 is initially projected down the incline, determine its (vector) acceleration.

9. A skier slides down a 150.0 ft ski jump ramp that makes an angle of 37° with the horizontal. The coefficient of sliding friction of waxed hickory skis on dry snow is about 0.050 (CRC Handbook of Chem. and Phys.). Neglect air resistance and determine the acceleration of the skier. If the skier starts from rest, what will be the skier's speed at the end of the ramp?

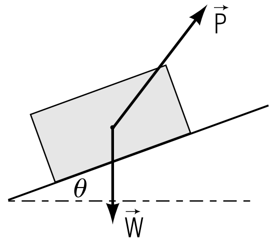
10. You are in your best car at the local dragway. The "christmas tree" (column of lights that "counts down" the start of a race) is not yet lit. The coefficient of non-sliding friction between your tires and the strip is 2.13, while the coefficient of sliding friction is 1.02 (courtesy of the CRC Handbook). The lights on the christmas tree are blinking down –yellow–yellow–GREEN! Do you:

a. stomp on the gas and then let back to where your wheels are just barely spinning as you accelerate, or

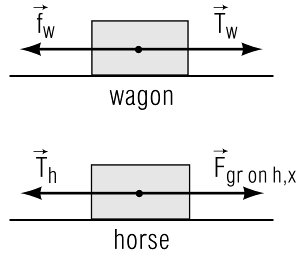
b. stay just below where your wheels begin to spin? Explain.

Brief Answers:

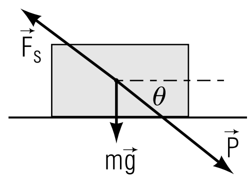
1a.



2a.

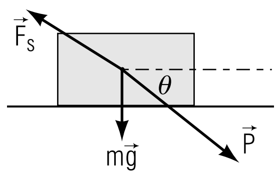


3a.

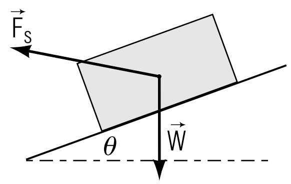


4a. (same as 3a)

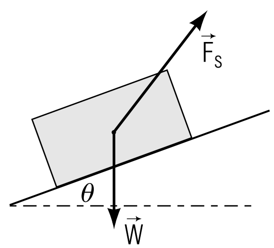
5a.



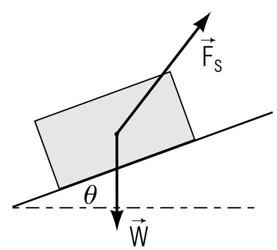
6a.



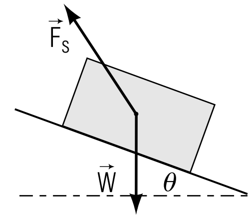
7a.



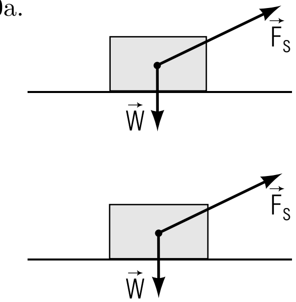
8a.



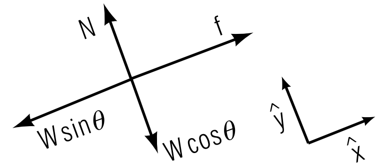
9a.



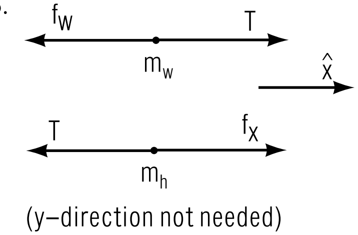
10a.



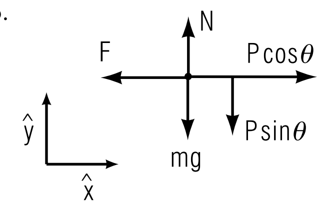
1b.



2b.

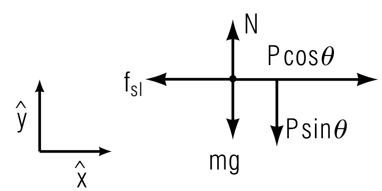


3b.

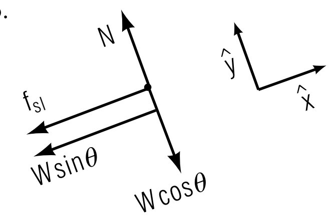


4b. (same as 3b)

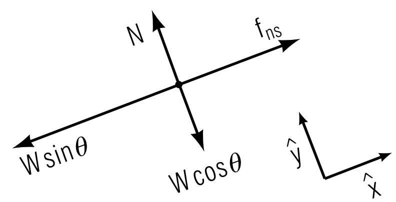
5b.



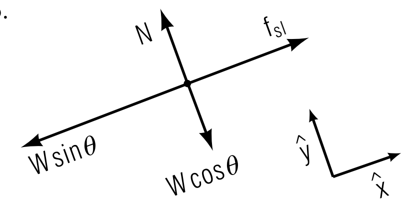
6b.



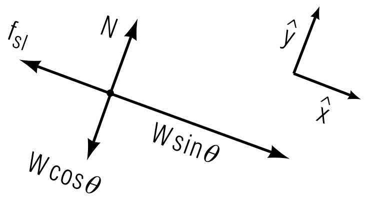
7b.



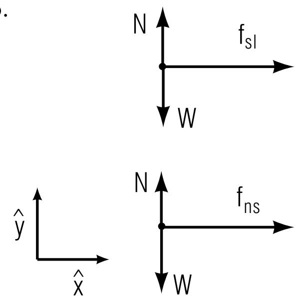
8b.



9b.



10b.



1c. $f - W\sin\theta = ma_x = (W/g)\,a_x$

$N - W\cos\theta = 0$

2c. $T - f_w = m_w\,a_x = (W_w/g)\,a_x$

$F_x - T = m_h a_x = (W_h/g)\,a_x$

3c. $p\cos\theta - f = 0$

4c. $p\cos\theta - f = 0$

$N - p\sin\theta - mg = 0$

$f = \mu_{ns}N$

5c. $p\cos\theta - f = ma_x$

$N - p\sin\theta - mg = 0$

$f = \mu_{sl}N$

6c. $-f_{sl} - W\sin\theta = ma_x = (W/g)\,a_x$

$N - W\cos\theta = 0$

$f_{sl} = \mu_{sl}N$

7c. $f_{ns} - W sin\theta = 0$ (for staying at rest)

$N - W\cos\theta = 0$

$f_{ns}(\text{max}) = \mu_{ns}N$

8c. $f_{sl} - W\sin\theta = ma_x = (W/g)\,a_x$

$N - W\cos\theta = 0$

$f_{sl} = \mu_{sl}N$

9c. $W\sin\theta - f_{sl} = ma_x = (W/g)\,a_x$

$N - W\cos\theta = 0$

$f_{sl} = \mu_{sl}N$

10c. (i) $f_{sl} = ma_x;\ W = N;\ f_{sl} = \mu_{sl}N$

(ii) $f_{ns} = ma_x;\ W = N;\ f_{ns} = \mu_{ns}N$

1d. $f = W\,[(a_x/g) + \sin\theta]$

$N = W\cos\theta$

$\vec{p} = W\,[(a_x/g) + \sin\theta]\hat{x} + W(\cos\theta)\hat{y}$

2d. $F_x = (W_w + W_h)\,(a_x/g) + f_w$

$T = W_w\,(a_x/g) + f_w$

3d. $f = p\cos\theta$

4d. $p = \mu_{ns}\,mg/(cos\theta - \mu_{ns}\,\sin\theta)$ *Help: [S-2]*

5d. $\mu_{sl} = (p\cos\theta - ma_x)/(p\sin\theta + mg)$

6d. $a_x = -g\,(\mu_{sl}\,\cos\theta + \sin\theta)$

7d. $f_{ns} = W\sin\theta;\ f_{ns}(\text{max}) = \mu_{ns}\,W\cos\theta$

8d. $a_x = g\,(\mu_{sl}\,\cos\theta - \sin\theta)$

9d. $a_x = g\,(\sin\theta - \mu_{sl}\,\cos\theta)$

10d. (i) $a_x = \mu_{sl}\,g$; (ii) $a_x = \mu_{ns}\,g$

1e. $f = (32000\,\text{lb})\,[\,(-1\,\text{ft/s}^2)/(32\,\text{ft/s}^2) + (\sin 2.87°)\,] = 6.0 \times 10^2\,\text{lb}$ *Help: [S-1]*

$N = (32000\,\text{lb})\,(\cos 2.87°) = 3.2 \times 10^4\,\text{lb}$

$\vec{p} = (6.0 \times 10^2\hat{x} + 3.2 \times 10^4\hat{y})\,\text{lb}$

2e. $F_x = (810\,\text{lb} + 1800\,\text{lb})\,(4.4\,\text{ft/s}^2)/(32\,\text{ft/s}^2) + 290\,\text{lb} = 6.5 \times 10^2\,\text{lb}$

$T = (1800\,\text{lb})\,(4.4\,\text{ft/s}^2)/(32\,\text{ft/s}^2) + 290\,\text{lb} = 5.4 \times 10^2\,\text{lb}$

3e. $f = (4.0\,\mathrm{N})\,(\cos 37°) = 3.2\,\mathrm{N}$

4e. $p = \dfrac{(0.30)\,(2.0\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{(0.80)\ -\ (0.30)(0.60)} = 5.88/.62 = 9.5\,\mathrm{N}$

5e. $\mu_{sl} = \dfrac{(12\,\mathrm{N})\,(.80) - (2.0\,\mathrm{kg})\,(2.0\,\mathrm{m/s^2})}{(12\,\mathrm{N})(.60) + (2.0\,\mathrm{kg})\,(9.8\,\mathrm{m/s^2})} = 0.21$

6e. $a_x = -(32\,\mathrm{ft/s^2})\,[\,(.30)\,(.94) + (.34)] = -2.0 \times 10^1\,\mathrm{ft/s^2}$;

(the negative sign means deceleration)

7e. $f_{ns} = (8.0\,\mathrm{lb})\,(0.34) = 2.7\,\mathrm{lb}$

$f_{ns}(\mathrm{max}) = (0.40)\,(8.0\,\mathrm{lb})\,(0.94) = 3.0\,\mathrm{lb}$

conclusion: required f_{ns} is less than $f_{ns}(\mathrm{max})$ so block will stay at rest.

8e. $a_x = (32\,\mathrm{ft/s^2})\,[\,(0.30)(0.94) - (0.34)] = -1.9\,\mathrm{ft/s^2}$; ("$-$"means down the incline)

9e. $a_x = (32\,\mathrm{ft/s^2})\,[0.60 - (0.050)\,(0.80)] = 17.9\,\mathrm{ft/s^2} \approx 18\,\mathrm{ft/s^2}$ (down the ramp)

$v = \sqrt{2ad} = [(2)\,(17.9\,\mathrm{ft/s^2})\,(150\,\mathrm{ft})]^{1/2} = 73 unit ft/s$

10e. $\mu_{ns} > \mu_{sl}$ so choice is (ii)

SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from PS-problem 1e)*

The problem of finding θ:

1. Draw the road surface and the horizontal line exactly as in the sketch that accompanies the statement of the problem.
2. Drop a vertical line from the upper end of the road surface you have drawn until the vertical line reaches the horizontal line. You have in front of you a right-angle triangle which has three sides.
3. Now mark the "5 feet" and the "100 feet" mentioned in the problem on the correct two sides of the triangle (pay close attention to the way those two numbers are described in the statement of the problem).
4. Mark "θ" on the correct angle in the triangle.
5. Now, knowing two sides of a right-angle triangle, you can find the angle. If you don't know how, see the trigonometry review in "Review of Basic Mathematical Skills" (MISN-0-401).

S-2 *(from PS-problem 4d)*

Look back at answer 4c and:

1. Solve the second equation for N.
2. Put that solution for N into the third equation wherever N occurs and then solve the resulting equation for f.
3. Put that solution for f into the first equation wherever f occurs and then solve the resulting equation for p.

MODEL EXAM

1. See Output Skill K1 in this module's *ID Sheet*.

Treat each problem as having five separate parts:

a. Draw a one-body diagram with the forces "as given" but with symbols representing all quantities;

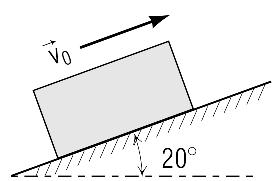
b. Draw a diagram similar to the one in part (a), but with the forces now represented only by their components, where one axis is along the direction of acceleration and the axes are labeled;

c. Write Newton's second law equations for the force components; and

d. Solve for the target unknown(s), still with all quantities represented by symbols.

e. Substitute numerical values along with their units and solve for the numerical answer(s). Express vector answers in terms of the unit vectors for the axes shown in part (b).

2. A block weighing 8.0 lb is projected up an incline of 20.0° with initial velocity v_0. For the interface between the block and the surface of the incline, $\mu_{ns} = 0.40$ and $\mu_{sl} = 0.30$. Determine the acceleration of the block.



3. When the block of Problem 2 stops, will it remain at rest or will it slide back down the incline? If it remains at rest, what frictional force will be acting on it?

4. If the block of Problem 2 is initially projected down the incline, determine its (vector) acceleration.

Brief Answers:

1. See this module's text.

2-4. See this module's *Problem Supplement*, Problems 6-8.