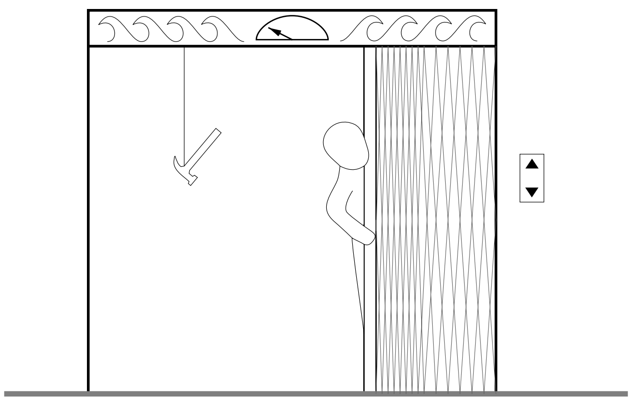
MISN-0-14

PARTICLE DYNAMICS; THE LAWS OF MOTION



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

PARTICLE DYNAMICS; THE LAWS OF MOTION

by

William Faissler

Title: **Particle Dynamics; The Laws of Motion**

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Input Skills:

1. Add and subtract vectors, both graphically and in terms of components (MISN-0-2).
2. Define position, velocity, and acceleration, and state the relationship between these quantities, both in one dimension (MISN-0-7) and in three dimensions (MISN-0-8).
3. Solve problems for one-dimensional motion with constant acceleration (MISN-0-7).
4. State Newton's three laws: explain the meaning of all terms, and give examples (MISN-0-62).

Output Skills (Knowledge):

K1. Tell the "Newton and the Horse Who Wouldn't Move" story and explain the flaw in the horse's reasoning.

Output Skills (Problem Solving):

S1. Solve problems involving single objects and constant forces. Some combination of the forces, mass of the object, and the acceleration will be given; you are to find the remaining terms. There are four possible levels of complexity in these problems:
   a. the forces all lie in one plane, hence graphical techniques can be used.
   b. the acceleration is not given directly; rather the kinematic relations for constant acceleration (two-step problems) will have to be used.
   c. The problem is truly three-dimensional, hence it is necessary to work with unit vectors or with component equations.
   d. a combination of (b) and (c).

S2. Solve "person in elevator" type problems.

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1. Introduction

1a. The Laws of Motion. This module provides a detailed introduction to Newton's three laws of motion. Newton's laws are rules that tell how an object's velocity and hence its energy and momentum can be changed. They tell under what circumstances an object's velocity is constant, when it will change, and what happens to whatever is causing it to change.

1b. How the Laws are Presented. Each law is stated and then illustrated with a simple idealized experiment. Such experiments are included to give you an intuitive feeling for the meaning of the laws. The use of each law is illustrated with an example. Further applications of the laws are given in the last section of the text.

1c. Why We Believe the Laws of Motion. It is important to realize that the laws of motion are not "proved" in the usual sense of the word and that they are most certainly not proved by the examples given in this module. We believe in the laws of motion–that is, we are willing to keep using them—because of the fact that all of the laws and definitions taken together make up a system that enables one to predict correctly what will happen in an experiment. Literally millions of such experiments have been done, both very simple ones and exceedingly complex ones, and within their range of applicability, these laws have always given the answers produced by the experiments.

2. Newton's First Law of Motion

2a. Introduction. It is common knowledge that a sliding object tends to slow down and eventually stop. The ancient Greeks felt this was easy to understand since they believed that the natural state of most objects was at rest; and in fact, they devoted considerable effort in attempting to explain how an object could keep moving when it was not being pushed. Since Newton's time, scientists have realized that the thing to explain is the cause of the acceleration, not the cause of the motion. Newton's first law of motion emphasizes this change in attitude by stating that there

must be a cause for an object's acceleration, while no cause is needed for constant velocity.

2b. Newton's First Law of Motion. Newton's first law is:

> A completely isolated object, that is, one subject to no external influences, has a constant vector velocity ($\vec{a} = 0$).

Thus an isolated object has no acceleration; if it is moving, it simply keeps on moving. No explanation is needed for motion; rather, an explanation is needed for acceleration, for a *change* in velocity. Of course, being at rest is a special case of moving with a constant velocity − a velocity of zero. Here is another way of stating the first law of motion: If an object is accelerating, then it is being acted on by an external influence. The following two paragraphs show that this law is not really contrary to common experience.

2c. Friction Causes Acceleration. A series of simple experiments shows that friction is one reason why many objects slow down. Consider a block sliding across a table. It is certainly true that unless you keep pushing it, it slows down and eventually stops. However, if you polish the surfaces, you will find that the block slides farther before it comes to a stop. If you oil or grease the surface lightly, you find that the block slides even farther before it comes to rest. If you use a very smooth surface and an object that "floats" on a cushion of air (such as a hovercraft), so that almost all friction is eliminated, you can get the object to slide incredible distances before it comes to rest. If the object is taken into space where there is essentially no friction and is given an initial velocity, as nearly as we can tell, it simply does not slow down, ever. Thus friction is one cause of acceleration (actually, deceleration).

2d. Other Causes of Accelerations. There are cases where an object undergoes an acceleration that is not caused by friction; however, in these cases it is possible to devise experiments to show that the acceleration is due to some other external influence acting on the object. One common example concerns the fact that objects fall; that is, they accelerate toward the earth. We interpret this as being due to the gravitational attraction that the earth exerts on the object.[1] Today we do experiments in space which show that this attraction gets weaker as an object gets farther from earth. From this we conclude that, far from the earth, an object would not fall toward the earth. (Newton was able to deduce this

[1] See "The Law of Universal Gravitation" (MISN-0-101).

from considerations of data on the solar system.) Thus we conclude that objects fall because of the influence of the earth. In a similar manner, it is possible to show that in every case in which an object undergoes an acceleration, the acceleration is due to some external influence acting on the object. This is just the first law of motion.

2e. The Use of the First Law of Motion. It is often stated that Newton's first law is just a special case of his second law, but this is not true. All kinematic and dynamic measurements must be made with respect to some system of reference. Only in certain systems of reference (nonaccelerated ones) will our laws of particle dynamics be found to hold.[2] Newton's first law gives a test to apply to a measuring system to verify that it is a suitable system in which to use our laws of particle dynamics. Observe an isolated object from your own reference system; if the object has a constant vector velocity, then you will find that all of particle dynamics holds in your reference system.

3. Newton's Second Law of Motion

3a. The Second Law of Motion. Newton's first law of motion states that an isolated object has no acceleration. His second law of motion gives the relationship between the acceleration of an object and the forces acting on it. The second law of motion is:

> The (vector) acceleration of an object is proportional to the (vector) force acting on it.

As an equation this is:

$$\vec{F} = m\,\vec{a}\,, \tag{1}$$

where m is the mass of the object. Thus, in the case of a single force applied to an object, if you know any two of the three quantities $\vec{F}$, m, or $\vec{a}$ you can immediately solve Eq. (1) for the third. Here is an illustrative exercise:

▷ *Exercise 1*: The wind exerts a force of 100 newtons in a northward direction on a sailboat having a mass of 250 kilograms. What is the acceleration of the boat? (Don't worry about the units of force and mass at

[2] See "Centripetal and 'g' Forces in Circular Motion" (MISN-0-17), "Classical Mechanics in Rotating Frames of Reference" (MISN-0-18), and "The Equivalence Principle" (MISN-0-110).

this time; they will be discussed in Sect. 5.) The acceleration will come out in m/s^2 for these units. *Help: [S-1]*[3]

3b. Mass. The mass of an object is an intrinsic property of the object. It is a scalar and is always positive. The usual way to find the mass of an object is to weigh it and calculate the mass from:

$$\begin{aligned} \text{Weight} &= \text{ mass } \times \text{ constant} \\ w &= m\,g\,. \end{aligned} \tag{2}$$

The value of g for Eq. (2) is given in the table below for several combinations of units. Mass and weight are discussed at length elsewhere.[4]

Units of Weight	Units of Mass	Value of g
newtons	kilograms	9.81 newtons/kilogram
pounds	kilograms	2.21 pounds/kilogram
pounds	slugs	32.2 pounds/slug
dynes	grams	981 dynes/gram

3c. Experimental Verification of Newton's Second Law. A number of simple experiments demonstrate that Newton's second law is consistent with both your intuitive notions and with nature. You know that if you push an object its velocity changes. By doing a simple experiment involving a block sliding on a more or less frictionless surface and using a spring scale, you can establish that the block accelerates in the direction of the force and that the acceleration is proportional to the force (see Fig. 1). You can easily devise a number of similar experiments involving pushes, pulls, and simple measuring devices. In all cases, you will find that the acceleration is proportional to the magnitude of the force as determined by the measuring device and that this magnitude corresponds roughly to your own physiological impression of "how hard" you have pushed or pulled the object. This is just what the second law says.

3d. A Statement of the Superposition Principle. If you do an experiment to investigate what happens when two or more forces are simultaneously applied to the same object, you will find that the acceleration of the object is the same as that caused by a single force equal to the vector sum of the individual forces being applied to the object. The

[3] The answers and detailed solutions to the exercises are given in this module's *Special Assistance Supplement* if you need help.

[4] See "Mass and Weight" (MISN-0-64) which provides a careful discussion of the definitions of weight and mass and of the differences between the two.

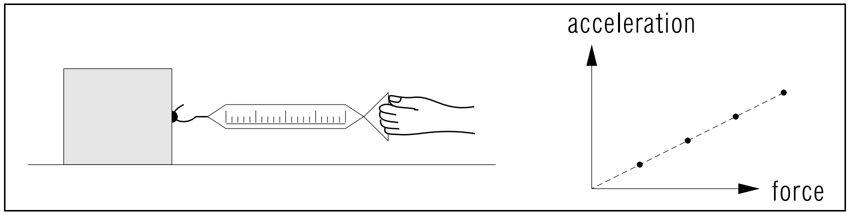

Figure 1. (a) An experimental arrangement for measuring the relationship between force and acceleration. (b) A typical result—the second law.

experimental result is often called the "superposition principle" for forces. A somewhat formal statement of it is:

> Two or more forces applied simultaneously to a particle have the same effect as a single force equal to the vector sum of the individual forces.

An example of this is illustrated in Fig. 2.

3e. A Restatement of the Second Law of Motion. Using the superposition principle, the second law of motion can be stated as:

> The acceleration of an object is proportional to the total vector force acting upon it.

As an equation:

$$\sum \vec{F} = m\,\vec{a}\,. \tag{3}$$

The summation $\sum$, pronounced "sigma," reminds you to take the vector sum of all the forces applied to the object. Here is an exercise that will give you a little practice in using this law:

▷ *Exercise 2.* Forces of 25unitnewtons north, 15 newtons east, and 40.0 newtons south are applied to a 5.0 kg object. Show that the object has an acceleration of 4.24 m/s^2 in a direction 45° south of east. *Help: [S-2]* [5]

[5]Additional help on the second law is given in Sect. 1a of this module's *Special Assistance Supplement*.

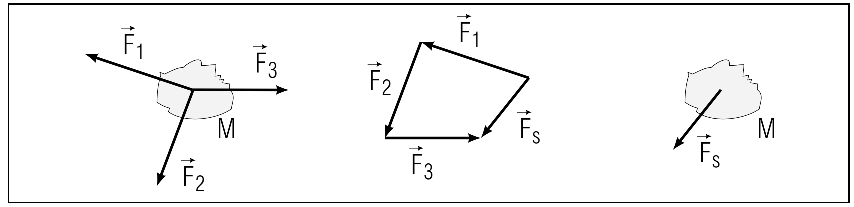

Figure 2. An example of the superposition of forces. The acceleration of the object acted upon by the three forces shown on the left is the same as the acceleration caused by $\vec{R}$, the vector sum of the three forces.

4. Newton's Third Law of Motion

4a. The Third Law of Motion. When two objects interact, such as when two billiard balls collide, each one of them exerts a force on the other. Newton's third law of motion relates the force the first object exerts on the second to the force the second exerts on the first. The law is:

> To every action there is always opposed an equal reaction: or, the mutual actions of two bodies upon each other are always equal and in opposite directions.

Putting this in other words:

> If object A exerts a force on object B, then object B exerts an equal but oppositely directed force on object A. Further, these forces both lie along the line joining the two bodies.

This law is sometimes casually quoted as "action equals reaction."

4b. A Demonstration of the Third Law. Figure 3 shows an experimental arrangement that is frequently used to demonstrate the third law of motion. Two identical blocks (or carts) are tied together by a string with a light spring compressed between them. The surfaces are very smooth, so the blocks slide very freely. The blocks are initially at rest, so according to the first law they remain at rest as long as the string is intact. Now, if the string is very carefully broken - it is traditional to burn it with a laser beam—the objects spring apart. BIG DEAL!!! However, careful measurements reveal two surprising (?) facts for two identical objects:

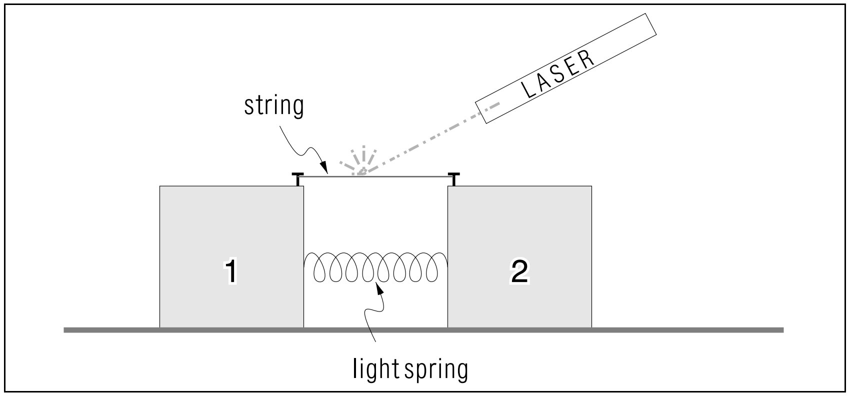

Figure 3. A demonstration of the third law of motion.

(a) The objects always move apart in the precisely opposite directions.

(b) The magnitudes of the two accelerations are always precisely the same.

Hence, when the acceleration is over, the velocities of the two identical objects are equal but opposite: $\vec{v}_1 = -\vec{v}_2$. This same result occurs for big objects and for small ones, for cases when the spring is lightly compressed and when it is strongly compressed, and for cases where the spring is set up to pull as well as to push, as long as the two objects are truly identical and are isolated from all other influences.[6] Since our two objects have the same mass and equal but opposite accelerations, then, according to the second law, the force the first block exerts on the second (via the spring) must be equal and opposite to the force the second block exerts on the first. This is the third law of motion.

4c. A Comment About the Third Law. It is very important to realize that although the third law says that forces always occur in pairs, each member of a pair always acts on a different object from that acted on by the other member of the pair. (If they were to act on the same body, then it would be impossible ever to accelerate a body, since the resultant force would always be zero!)

[6]The case where the two objects are not identical is treated in "Momentum: Conservation and Transfer" (MISN-0-15).

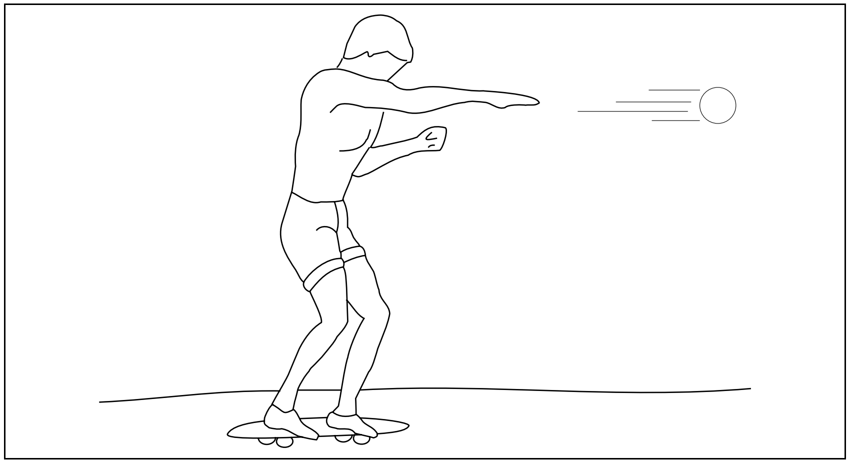

Figure 4. A novel way to propel a skateboard.

4d. Two Simple Examples of the Third Law. As a simple example, imagine that you are hitting a tennis ball. Your racket exerts a force on the ball, causing it to accelerate. At the same time, the ball exerts an equal but opposite force on the racket, causing it to accelerate in the opposite direction (slowing it down a bit). Thus the action and reaction act on different bodies and cause each body to be accelerated (but oppositely).

When you throw an object, such as a ball, you must exert a force on it to accelerate it. In return, the ball exerts an equal force on you in the opposite direction. As a result you "recoil" in the opposite direction from which you threw the ball. This recoil is particularly noticeable when you throw a very heavy object, or when you are standing on a slippery surface–such as a sheet of ice. Also, see Fig. 4.

5. Systems of Units

5a. Introduction. In order to use the laws discussed in this module, it is necessary to have all relevant quantities, (distance, time, mass and force) in a consistent set of units. The set of consistent units that is in use throughout most of the world is the SI (*International System*) of the ISO (*International Organization for Standardization*). The U.S. Government has put the SI units on the Web at the NIST (National Institute for Standards and Technology) site for *Constants, Units, and Uncertainty*:

http://physics.nist.gov/cuu/. The so-called *English system*, based on pounds, feet, etc., is in partial use in the U.S. and a few other countries. An older "cgs" system, based on the centimeter, gram, and second, is still used occasionally.

5b. SI Units. Using the second law of motion, you can see that the dimensions of force are

$$\text{mass} \times \text{length} \times \text{time}^{-2}\,.$$

In the SI system of units, length is measured in meters, mass in kilograms, and time in seconds. Thus force has units of

$$\text{kilogram meter second}^{-2}\,.$$

This quantity occurs so often that it has been given the name "newton." Thus:

$$\begin{aligned}\text{newton} &= \text{kilogram meter second}^{-2}\\ &= \text{kg}\,\text{m}\,\text{s}^{-2}\,.\end{aligned}$$

The abbreviation for both newton and newtons is N. To get a rough idea of how large one newton of force is, just remember that a quart of milk has a weight of roughly ten newtons.

5c. The CGS System. In the old cgs (centimeter-gram-second) system, length is measured in centimeters, mass in grams, and time in seconds. The unit of force is the dyne, defined as:

$$\begin{aligned}\text{dyne} &= \text{gram centimeter second}^{-2}\\ &= 1\,\text{g}\,\text{cm}\,\text{s}^{-2}\,.\end{aligned}$$

A dyne is a very small unit of force. The weight of a nickel is roughly 5,000 dynes, so a one-dyne force is a very, very weak force.

5d. The English System. In the English system of units, mass is measured in slugs, length in feet, and time in seconds. The unit of force is the poundand it is defined as:

$$\text{pound} = \text{slug foot second}^{-2}\,.$$

You are already aware of how strong a one-pound force is, but you are probably unacquainted with the slug. An object having a *mass* of one slug has a *weight* of roughly 32 pounds.

5e. A Table of Names of Units. The table below gives you a brief summary of these units and their abbreviations ("abb").

	SI units		cgs system		English system	
Quantity	Unit	abb	Unit	abb	Unit	abb
length	meter	m	centimeter	cm	foot	f
time	second	s	second	s	second	s
mass	kilogram	kg	gram	gm*	slug	(none)
force	newton	N	dyne	dyn	pound	lb

* Note that in SI units the gram is a derived quantity and its abbreviation is "g" (use context to tell when "g" means "gram" and when it means the gravitational constant.)

You can find the definitions of these systems and many useful conversion factors in any standard reference on physical quantities.[7]

6. More Applications

6a. Introduction. This section contains several more applications of the laws of motion to practical situations. Additional applications will be found in this module's Problem Supplement. Applications of the laws of motion to more complex situations and to situations involving two or more objects are found elsewhere.[8]

6b. An Object at Rest. The net force acting on an object at rest must be zero. As an example, consider a motionless ball sitting on a table. Since we have said that the ball remains at rest, it has no acceleration. According to the second law, this means that the total (net) force acting on it is zero. This does not mean that there are no forces acting on the ball; it does mean that the vector sum of all the forces acting on it is zero. In fact, for the ball resting on a table we know there are at least two forces acting on it: its weight is acting downward and the table is exerting an upward force on the ball (these forces are shown in Fig. 5). If

[7] See the *CRC Handbook of Physics and Chemistry*, Chemical Rubber Publishing Co., Cleveland, Ohio.

[8] To see the full power of the laws of motion you must apply them in practical situations. Many interesting situations are treated in "Applications of Newton's Second Law, Frictional Forces" (MISN-0-16). Why is it so hard to put a large space station into orbit around the earth? This problem is treated in "Mass Changing With Time: Rockets and Other Examples" (MISN-0-19).

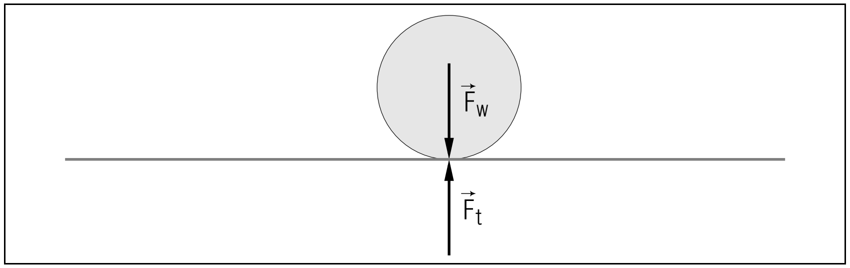

Figure 5. The forces acting on a ball at rest.

these are the only two forces acting on the ball, we have, successively:

$$\vec{F}_t + \vec{F}_w = 0\,,$$
$$\vec{F}_t = -\,\vec{F}_w\,.$$

> In general, for any object at rest, you immediately know that the vector sum of all the forces acting on it is zero.

In any specific case, you will have to work out how many and what forces are acting on an object of interest that is at rest. Then knowledge of all but one force will enable you to calculate that last force.

6c. An Object Moving at a Constant Velocity. When an object is moving at a constant (vector) velocity, the net force acting on it must be zero, just as for an object at rest. An airplane moving at a constant velocity has no acceleration, hence we must get zero for the vector sum of all the forces acting on the plane: the weight of the plane, plus the lift due to the wings, plus the push due to the engine, plus the air drag–see Fig. 6. In the notation defined in Fig. 6, this result is:

$$\vec{F}_D + \vec{F}_P + \vec{F}_L + \vec{F}_W = 0\,.$$

Since being at rest is just a special case of moving at a constant velocity—here with a constant velocity of zero, it should be no surprise that you know the same amount about the forces acting on the object as you do in the ball-at-rest situation, except that in the case of the airplane the forces may appear to be more readily identifiable.

6d. "Person in Elevator" Problem. A very common type of problem involves an object (a person, a light bulb, a monkey, etc.) that is hung by a string from the ceiling of an elevator (Fig. 7). The problem

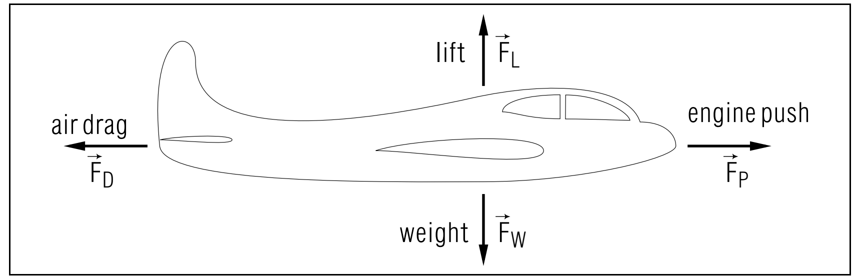

Figure 6. The forces acting on an airplane.

is to calculate the tension in the string and find how it depends on the acceleration of the elevator.

A Qualitative Discussion:
This problem can be solved by a straightforward application of the second law of motion. There are only two forces acting on the object: its weight acting downward and the upward pull of the string; these forces are shown in Fig. 8.

The object accelerates with the elevator. Thus if the elevator accelerates upward, so does the object. But if the object is accelerating upward, then according to the second law the net force on the object must be upward. This means that the upward pull must be greater than the downward weight.

$$F_p > F_w; \qquad \text{accelerating upward}\,.$$

If the elevator is not accelerating, neither is the object and there is no net force on the object. Thus the pull must be equal to the weight.

$$F_p = F_w; \qquad \text{no acceleration}\,.$$

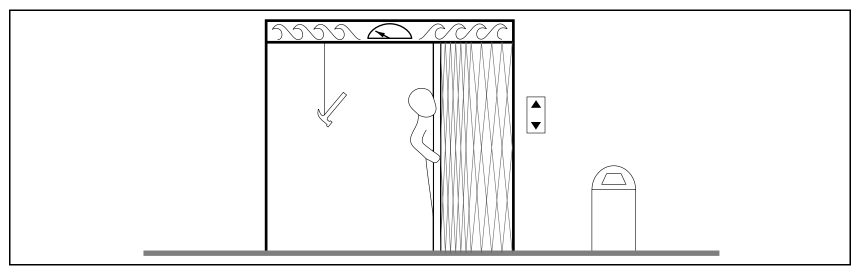

Figure 7. A hammer hung from an elevator ceiling.

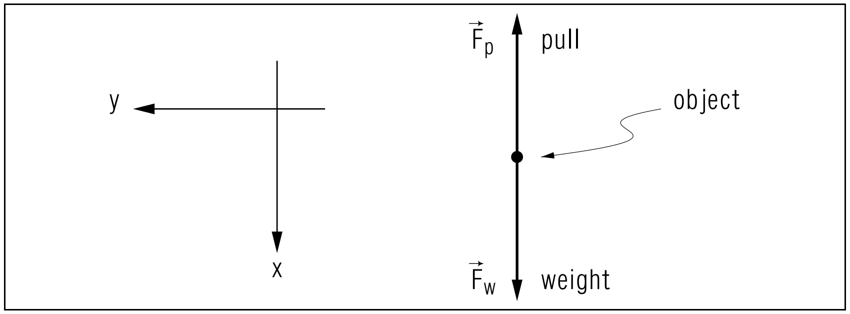

Figure 8. Forces acting on an object hung in an elevator.

If the elevator is accelerating downward, then so is the object. This means that the net force on the object must be downward, so the weight must be greater than the upward pull.

$$F_p < F_w; \qquad \text{accelerating downward}\,.$$

(The problem becomes more complicated if the elevator accelerates downward faster than the acceleration of free fall, $9.8\,\mathrm{m}/s^2$, but we will assume that this does not happen here.

The Qualitative Solution:
The expression for the pull of the string can be found by picking a coordinate system and using the second law of motion. We pick a coordinate system with the x axis downward as shown in Fig. 8. Thus a downward acceleration is positive and an upward acceleration is negative. The second law of motion is:

$$\sum \vec{F} = m\,\vec{a}\,.$$

The x component of the weight is $+F_w$ while the x component of the pull is $-F_p$ (it is upward). Hence the x component equation is:

$$F_w - F_p = m\,a.$$

Solving for F_p we obtain.

$$F_p = F_w - m\,a; \qquad (\hat{x} \text{ taken downward})\,.$$

Check over the sign conventions. Then see if you agree that, if the positive x direction were upward, you would obtain:

$$F_p = F_w + m\,a; \qquad (\hat{x} \text{ taken upward})\,.$$

Variations on This Problem:
In one variation on the problem, the string is replaced by a spring scale, which then reads the apparent "weight" of the object as the elevator accelerates. If the acceleration of the elevator happens to be $9.8\,\mathrm{m/s^2}$ downward, in "free fall," then the object appears to be weightless. In other variations, the object is placed on the floor; this is essentially the same problem, only the pull of the string is replaced by the upward push of the floor, etc.

6e. The Horse. The third law of motion says that if you push on the wall, the wall pushes on you with an equal push in the opposite direction. Thus there are two equal and opposite forces here, but only one of them is on you. Be sure you understand this point. Discuss it with others if you are unsure about it. Any confusion here will surely result in incorrect problem solutions later. Try your understanding on this problem:

There is a famous tale often used to illustrate the third law. There was an iceman's horse who read Newton's writings (in Latin, yet) while his master was making deliveries. One day, when the iceman returned, the horse refused to try to move since he knew it would do no good. He reasoned as follows:

> "The second law says
>
> $$\sum \vec{F} = m\,\vec{a}\,.$$
>
> The third law says that for every action there is an equal and opposite reaction. Thus if I pull on the wagon, the wagon pulls equally hard on me in the opposite direction. Hence the vector sum of these two forces is zero and there is no acceleration. Hence I cannot get the wagon to start moving."

The horse was adamant; no amount of beating would make it go. Finally Newton was called in and he quickly persuaded the horse to move on, pulling its wagon. How? Can you convince the horse? *Help: [S-10]*

Acknowledgments

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PROBLEM SUPPLEMENT

These problems are grouped by skill so that you can easily find the ones corresponding to the skills for which you are responsible. The answers are at the end of the list. The notation " *Help: [S-n]*" refers to panel "*n*" in Part 3 of this module's *Special Assistance Supplement*. Problems 15 and 16 are also on this module's *Model Exam*.

Skill 1a (Problem-Solving):

1. Forces of 65 N north and 5.0×10^1 N east are applied to a 2.0 kg can of soup. Determine the acceleration of the can. *Help: [S-12]*

2. An 8.0×10^1 kg parachutist is "falling" with a downward acceleration of $1.5\,\mathrm{m/s^2}$. What is the magnitude of the upward force on the parachutist due to the parachute? *Help: [S-3]*

3. A modern small car having a weight of 3.0×10^3 pounds is accelerating at $5.0\,\mathrm{ft/s^2}$. What is the magnitude of the total force acting on it? *Help: [S-4]*

Skill 1b (Problem-Solving):

4. An 8.0×10^3 kg meteorite slashes through the sky, slams into the earth at 4.0×10^2 m/s and comes to a stop 16 m below ground level. Assuming the meteorite was decelerating uniformly while burrowing through the ground, what was the magnitude of the net force on it during those times? *Help: [S-5]*

5. An irate college student on a bicycle is bearing down on a panic-stricken pedestrian. At a distance of 6.0 m from the pedestrian, she slams on her brakes and slides uniformly to a stop in 1.5 seconds. What was magnitude of the average stopping force? Is the pedestrian bicycle-struck? The total mass of the bicycle and operator is 55 kg; her initial velocity was 8.8 m/s. *Help: [S-6]*

6. A 1,950 pound Volkswagon accelerates from 0 to 6.0×10^1 miles per hour (88 ft/s) in 45 sec. What is the magnitude of the average force on it? *Help: [S-14]*

7. A "souped-up" Volkswagon, weighing 2020 lb., reaches a speed of 45 miles per hour (66 ft/s) in a distance of 1320 feet after starting from rest. What is the magnitude of the average force on the car? *Help: [S-14]*

8. Three forces;
$$\vec{F}_1 = (3.0\hat{x} + 4.0\hat{y} - 5.0\hat{z})\,\mathrm{N}$$
$$\vec{F}_2 = (6.0\hat{x} + 6.0\hat{y} + 6.0\hat{z})\,\mathrm{N}$$
$$\vec{F}_3 = (21.0\hat{x} - 16.0\hat{y} + 17.0\hat{z})\,\mathrm{N}$$
are applied to a 6.0 kg object. Find its acceleration. *Help: [S-7]*

9. Rod Carew hits a baseball from a practice tee. The bat exerts a force of 55 N on the ball and the ball accelerates with
$$\vec{a} = (350\hat{x} + 150\hat{y})\,\mathrm{m/s^2}.$$
Find the mass of the ball. *Help: [S-13]*

10. A large athletically inclined student applies two forces,
$$\vec{F}_1 = (3.0\hat{x} + 15.0\hat{y})\,\mathrm{N}$$
$$\vec{F}_2 = (16.0\hat{x} - 25.0\hat{y})\,\mathrm{N}$$
to a 44.0 kg (98 pound) lab instructor. What is the magnitude of the acceleration of the instructor?

Skill 1c (Problem-Solving):

11. An electron, mass 9.1×10^{-31} kg, has an initial velocity of:
$$\vec{v}_1 = (3.0 \times 10^6\hat{x} + 2.0 \times 10^6\hat{y} + 0\hat{z})\,\mathrm{m/s}.$$
After it has undergone 5.0 seconds of constant acceleration, its velocity is:
$$\vec{v}_f = (0\hat{x} + 5.0 \times 10^6\hat{y} + 1.0 \times 10^7\hat{z})\,\mathrm{m/s}.$$
Find the magnitude of the force applied to it. *Help: [S-8]*

12. A 5.0 kg mass has an initial velocity of $(3.0\hat{x} + 6.0\hat{y})$ m/s. Sixteen seconds later it has a velocity of $(9.0\hat{x} - 9.0\hat{y})$ m/s. Assuming the acceleration was constant, what were the x and y components of the force?

Skill 2 (Problem-Solving):

13. A bag of corn is placed on a spring scale in an elevator. When the elevator is at rest, the scale reads 8.0×10^1 lb. What does the scale read when:

    a. the elevator accelerates upward at $8.0\,\mathrm{ft/s^2}$?

    b. the elevator moves upward with a constant velocity of $10.0\,\mathrm{ft/s}$?

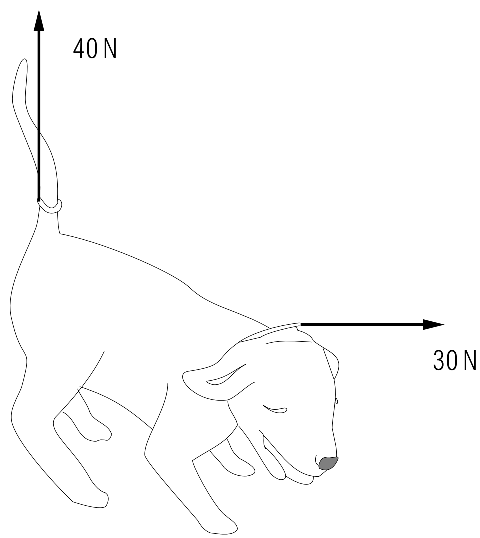
    c. the elevator accelerates downward at $5.0\,\mathrm{ft/s^2}$? *Help: [S-9]*

14. A 5.0 kg light fixture is hung from the ceiling of an elevator by a cord having a breaking strength of 65 N. What is the maximum upward acceleration the elevator can have without the cord breaking?

At Large:

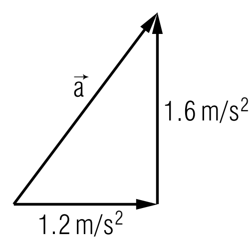
15. A 5.0 kg ham is hanging from a spring scale attached to the ceiling of an elevator that is accelerating upward at $3.0\,\mathrm{m/s^2}$. What does the scale read as the apparent weight of the ham?

16. The components of the *net* force on a 25 kg dog are shown. Sketch the direction of the acceleration of the dog and calculate the magnitude of the acceleration.



Brief Answers:

1. $32.5\,\mathrm{m/s^2}$ north, $25\,\mathrm{m/s^2}$ east, or $|\vec{a}| = 41\,\mathrm{m/s^2}$ at $\theta = 52°$ north of east. *Help: [S-12]*
2. 6.6×10^2 N.
3. 4.7×10^2 lb.
4. 4.0×10^7 N.
5. 3.2×10^2 N; The pedestrian will be struck.
6. 1.2×10^2 lb.
7. 1.0×10^2 lb. *Help: [S-14]*
8. $\vec{a} = (5.0\hat{x} - 1.0\hat{y} + 3.0\hat{z})\,\mathrm{m/s^2}$; $|\vec{a}| = 5.9\,\mathrm{m/s^2}$.
9. 144 grams. *Help: [S-13]*
10. $|\vec{a}| = 0.49\,\mathrm{m/s^2}$.
11. $|\vec{F}| = 2.0 \times 10^{-24}$ N.
12. $x: 1.9$ N, $y: -4.7$ N.
13. (a) 100 lb. *Help: [S-9]*; (b) 80 lb. *Help: [S-9]*; (c) 68 lb. *Help: [S-9]*
14. $3.2\,\mathrm{m/s^2}$.
15. 64 N.
16. $|\vec{a}| = 2.0\,\mathrm{m/s^2}$; Sketch shows $a_x\hat{x}$, $a_y\hat{y}$, and $\vec{a}$.



SPECIAL ASSISTANCE SUPPLEMENT

PURPOSE. This part of the module provides you with additional assistance in mastering the concepts presented in the text and in applying them to the problems. You need not study this material if you feel you have mastered the material in the text and if you can do the problems.

CONTENTS.

PART 1: The Second Law of Motion (Text, Section 3). The following drill is intended to help sharpen your understanding of the second law of motion. To get the most benefit from it, first write your answers to a question and then check against the answers that follow all questions. Go back to the text and review it until you feel you understand any questions you got wrong. If you write your answers on scrap paper, you can use the drill several times until you get it perfect.

1. Write the Second Law of Motion.
2. In the second law, the $\vec{F}$ stands for ______________. This is a (vector/scalar) quantity.
3. The m stands for ______________. It is a (vector/scalar) quantity. It may take on both positive and negative values. (T/F)
4. The $\vec{a}$ stands for ______________. It is a (vector/scalar).
5. The second law is a (vector/scalar) equation.
6. Vector equations involve both ______________ and ______________.
7. The second law says nothing about the direction of the acceleration. (T/F)
8. The second law says that the acceleration of an object is proportional to any force you find is being applied to it. (T/F)

9. The magnitude of the acceleration of an object can never be greater than the magnitude of the total force applied to the object. (T/F or inappropriate?)
10. The mass times the acceleration is equal to the ______________ force applied to the object.
11. In adding all the forces together, you must use ______________ addition. This means that you do not need to worry about the directions of the individual forces. (T/F)
12. What is the relation between the x-component of the acceleration and the y-component of the force on the object?

Answers.

1. $\sum \vec{F} = m\vec{a}$.
2. $\vec{F}$ is a force applied to the object. Force is a vector.
3. m is the mass of the object whose motion is being considered. Mass is a scalar quantity and is always positive.
4. $\vec{a}$ is the acceleration of the object. Acceleration is a vector.
5. It is a vector equation.
6. Vectors involve both magnitude and direction.
7. False; vectors involve direction.
8. False; the acceleration is proportional to the total vector sum of all the forces acting on the object.
9. Inappropriate. The magnitudes of these quantities include units, so the quantities are not comparable. Their units are different.
10. Mass times acceleration equals total or net force applied to the object.
11. Use vector addition. False; vector addition involves direction.
12. None, as far as the laws of motion are concerned.

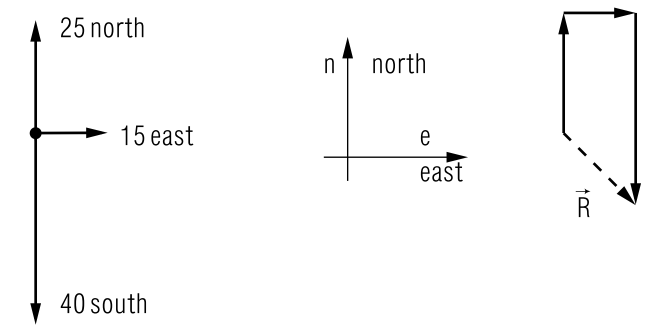
PART 2: Solutions to Exercises.

S-1 *(from TX-3a)*

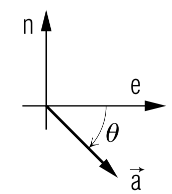
For the case of the single force applied to an object, the acceleration can be found by using Eq. (1). Thus we have: $\vec{a} = \vec{F}/m$.
The acceleration will be in the direction of the force, which in this case is northward. The magnitude of the acceleration can be found from the equation above: $|\vec{a}| = 100\,\mathrm{N}/250\,\mathrm{kg} = 0.40\,\mathrm{m/s^2}$.

S-2 *(from TX-3e)*

When several forces are applied to an object, you must add the forces using vector addition and then divide by the mass to find the acceleration. The sketch below shows the situation and a suitable coordinate system.



Since the forces are at right angles, by picking the coordinate axes to be north and east you eliminate the problem of finding the components of the forces. The second law is: $\vec{R} = \Sigma\,\vec{F} = m\,\vec{a}$. The northward component of $\vec{R}$ is then $R_n = (25 - 40)\,\mathrm{N} = -15\,\mathrm{N}$, and the northward component of $\vec{a}$ is $a_n = R_n/m = -15/5\,\mathrm{m/s^2} = -3\,\mathrm{m/s^2}$.

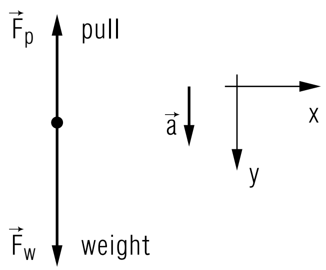


The eastward component of R is $R_e = 15\,\mathrm{N}$, and the eastward component of $\vec{a}$ is $a_e = R_e/m = 15/5 = 3\,\mathrm{m/s^2}$. The magnitude of $\vec{a}$ is then: $|\vec{a}| = \left(a_n^2 + a_e^2\right)^{1/2} = 4.243\,\mathrm{m/s^2}$, and the direction of $\vec{a}$ is given by the angle θ defined in the sketch with: $\tan\theta = a_n/a_e = -1$; $\theta = 45°$ south of east.

PART 3: Solutions to Selected Problems.

To get maximum benefit from the practice problems and from this section, you should have made a determined effort to solve the problems yourself before you turn to this section for "special assistance."

S-3 *(from PS-problem 2)*



This problem is very similar to the elevator problem. Begin by drawing a sketch showing all the forces and other relevant data. Pick the y axis as being downward. Since all the forces and accelerations are in the y direction, you can treat the second law as a scalar equation. Watching the signs, you have:

$$\Sigma F = m\,a$$
$$F_w - F_p = m\,a$$

Since the weight of the man is given by $F_w = mg$, this becomes: $F_p = F_w - ma = mg - ma = m(g - a) = (80\,\text{kg})(9.8 - 1.5)\,\text{m/s}^2 = F_p = 664\,\text{N}$.

S-4 *(from PS-problem 3)*

The only problem here is to remember that you have been given the weight of the object and you need the mass when you use Newton's second law. The mass and weight are related by $w = mg$, and using the constant given in the table in Sect. 3b, this gives: $m = 93.17\,\text{slug}$ (NOT 93.75) for the car.

S-5 *(from PS-problem 4)*

To find the force, you need to know the acceleration. You find the acceleration from the kinematic equations for "motion with constant acceleration starting (or ending) at rest":[a]

$$v_0 = \sqrt{2as}$$
$$a = \frac{v_0^2}{2s} = \frac{400^2}{2(16)}\,\text{m/s}^2 = 5000\,\text{m/s}^2$$

The second law then gives

$$F = ma = (8000)(5000)\,\text{kg m/s}^2 = 4.0 \times 10^7\,\text{N}.$$

[a] See "Kinematics in One Dimension" (MISN-0-7).

S-6 *(from PS-problem 5)*

Again, to find the force, you must first find the acceleration. For constant acceleration you know: $v = v_0 + a\,t$. For $v = 0$ this gives:

$$a = -\frac{v_0}{t} = -\frac{8.8}{1.5}\,\text{m/s}^2 = -5.9\,\text{m/s}^2\,.$$

The magnitude of the force is then given by

$$F = |ma| = 3.2 \times 10^2\,\text{N}\,.$$

While stopping, the bicycler goes a distance s given by:

$$s = v_0 t + \frac{1}{2}at^2 = (8.8)(1.5) - (0.5)(5.8667)(1.5)^2 = 6.6\,\text{m}.$$

The poor pedestrian has had it.

S-7 *(from PS-problem 8)*

This is a simple exercise in adding vectors through the use of unit vectors. If you have difficulty with this, review the unit on vector addition.[a] With no further comment, here are several intermediate steps in a solution to this problem:

$$\begin{aligned} m\,\vec{a} &= \Sigma\,\vec{F} \\ &= (3\hat{x} + 4\hat{y} - 5\hat{z} + 6\hat{x} + 6\hat{y} + 6\hat{z} + 21\hat{x} - 16\hat{y} + 17\hat{z})\,\mathrm{N} \\ m\vec{a} &= (30\hat{x} - 6\hat{y} + 18\hat{z})\,\mathrm{N}. \\ \vec{a} &= (5\hat{x} - 1\hat{y} + 3\hat{z})\,\mathrm{m/s^2} \\ |\vec{a}| &= 5.9\,\mathrm{m/s^2}. \end{aligned}$$

Further Help: [S-13]

[a] "Sums, Differences and Products of Vectors" (MISN-0-2).

S-8 *(from PS-problem 11)*

To solve this you need to use $\vec{F} = m\vec{a}$ and $\vec{v}_f - \vec{v}_i = \vec{a}\Delta t$, (valid for constant $\vec{a}$).
Combining these we get:

$$\vec{F} = \frac{m}{\Delta t}(v_f - v_i)$$

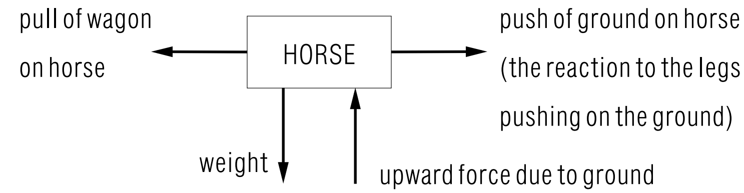
The unit vectors simply aid in the vector addition. You can work out the rest of the details.

S-9 *(from PS-problem 13)*

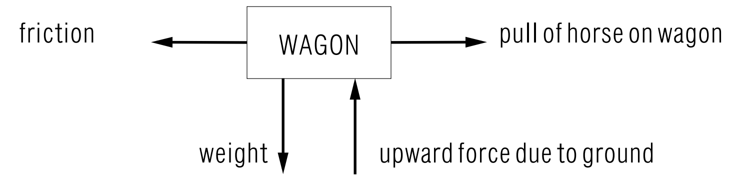
The only trick here is to recognize that the spring scale "reads" the force being applied to the bag. In each case draw a sketch, pick a coordinate system in which to determine the signs of the various terms, and proceed as in the example in the text.

S-10 *(from TX-6e)*

the forces on the horse:



the forces on the wagon:



The sketch shows all the forces on the horse and on the wagon.
It certainly is true that the third law says the pull of the horse on the wagon is equal and opposite to the pull of the wagon on the horse. Further, the second law says the vector sum of all the forces on the wagon equals the mass of the wagon times its acceleration.
However, the real question is: can the horse exert an unbalanced force on the wagon? The pull of the wagon on the horse is irrelevant because it is a force on the horse, not a force on the wagon.
For further analysis, see [S-11].

S-11 *(from [S-10])*

The only forces acting on the wagon are:

1. The pull of the horse.
2. The friction in the wagon axles and between wagon wheels and the ground.
3. The weight of the wagon (the gravitational force on it).
4. The upward force of the ground on the wheels.

The horse need only pull harder than the frictional force in order to accelerate the wagon horizontally, to give it a non-zero horizontal velocity.

S-12 *(from PS-problem 1)*

Here are the reminders consultants say they have given students who were unable to work this problem correctly or who were troubled by the problem's wording:

- kg is a metric unit of mass, not weight.
- weight = mass × g, whether in the English system or the metric.
- An object can be described in terms of its mass, weight, color, etc. Thus it is just fine to speak of a "six pound baby" or a "three kilogram cannister" or a "red car" or, as in this problem, a "2.0 kg can of soup."

S-13 *(from PS-problem 9)*

For how to obtain the magnitude of a vector, see module 2.

S-14 *(from PS-problem 7)*

$F_{\rm ave} = m\, a_{\rm ave}$ (always)

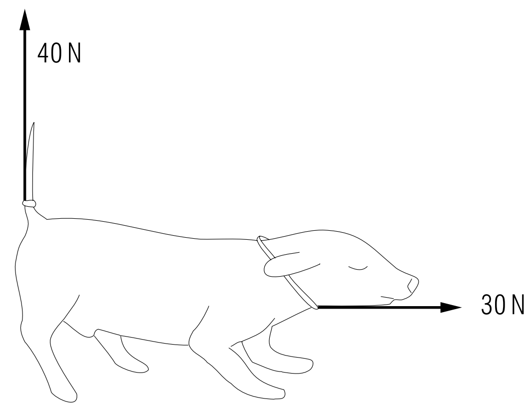
$a_{\rm ave} = \dfrac{\Delta v}{\Delta t}$ (always)

Make sure you successfully work problem 4 before attempting this problem.

Also, see [S-12].

MODEL EXAM

1. See Output Skill K1 in this module's *ID Sheet*.
2. A 5.0 kg ham is hanging from a spring scale attached to the ceiling of an elevator that is accelerating upward at $3.0\,{\rm m/s^2}$. What does the scale read as the apparent weight of the ham?
3. The components of the net force on a 25 kg dog are shown. Sketch the direction of the acceleration of the dog and calculate the magnitude of the acceleration.



Brief Answers:

1. See this module's text.
2. See this module's *Problem Supplement*, problem 15.
3. See this module's *Problem Supplement*, problem 16.